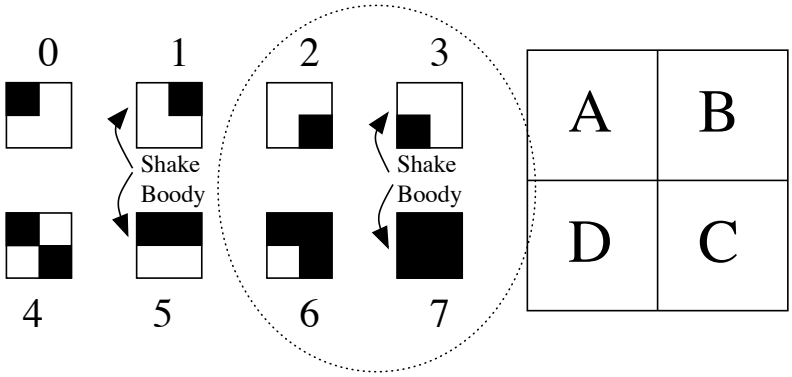


Wave hands

5. (25 pts) (Previous midterm question) Professor Rubenstein is thinking about opening up a disco. His groovy lighted floor will produce the patterns shown above while his patrons dance the hustle and do the smurf.

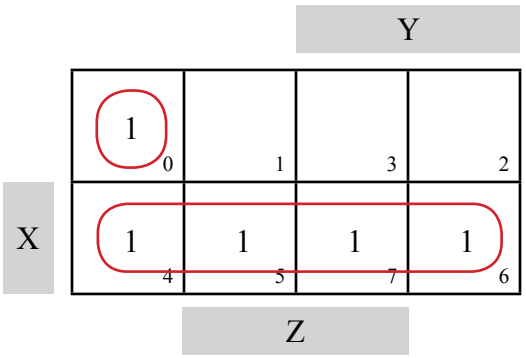
   (a) (10 pts) Given a **3-to-8 decoder** and as many 2-input **OR gates** as you want, design a circuit that takes in as input the binary value of a number between 0 and 7, and generates 4 outputs, A, B, C, and D. The outputs should equal 1 when the square that corresponds to that output is darkened. (It can be done with 7 OR gates).

   ANSWER:

   Find simplified expressions for A, B, C, D.

A = m0 + m4 + m5 + m6 + m7

$= \overline{X}\,\overline{Y}\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,\overline{Y}\,Z + X\,Y\,\overline{Z} + X\,Y\,Z$

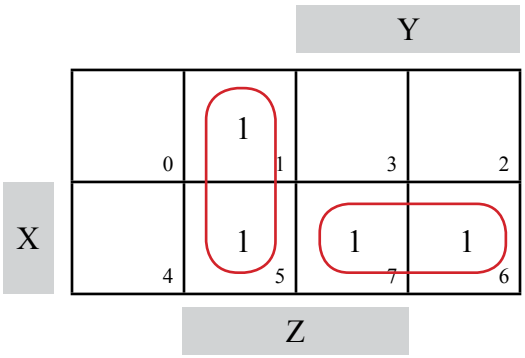


After simplifying using a Karnaugh map, we find that

$A = X + \overline{X}\,\overline{Y}\,\overline{Z}$

B = m1 + m5 + m6 + m7

$= \overline{X}\,\overline{Y}\,Z + X\,\overline{Y}\,Z + X\,Y\,\overline{Z} + X\,Y\,Z$

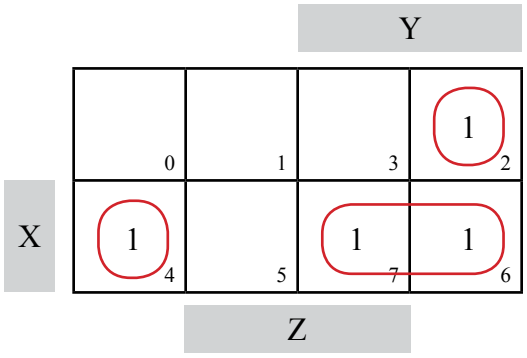


After simplifying using a Karnaugh map, we find that

$B = X\,Y + \overline{Y}\,Z$

C = m2 + m4 + m6 + m7

$= \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,Y\,\overline{Z} + X\,Y\,Z$

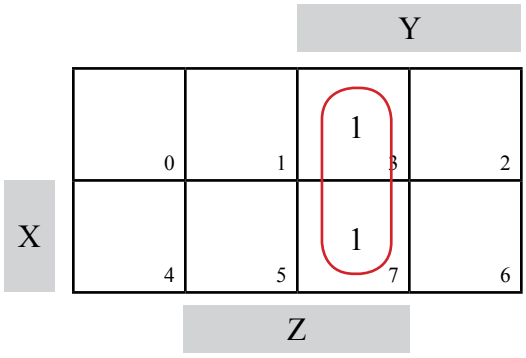


After simplifying using a Karnaugh map, we find that

$C = X\,Y + X\,\overline{Y}\,\overline{Z} + \overline{X}\,Y\,\overline{Z}$

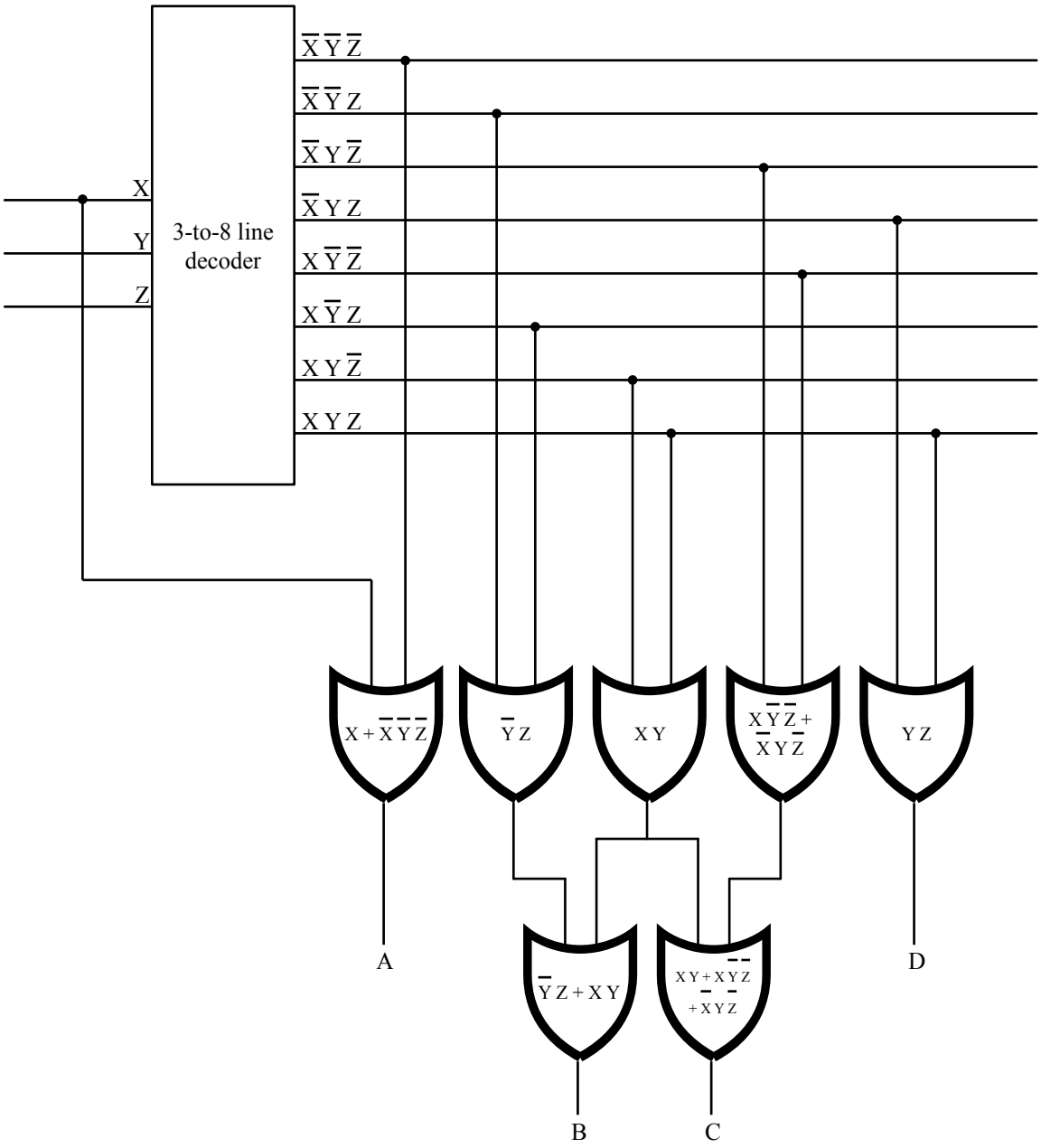
D = m3 + m7

$= \overline{X}\,Y\,Z + X\,Y\,Z$



After simplifying using a Karnaugh map, we find that

$D = Y\,Z$



(b) (15 pts) Professor Rubenstein also wants his disco to accommodate engineering-savvy individuals who aren't disco experts. Considering the 3-bit representation of the square number, when the middle bit (whose value is 2) is set (i.e., squares 2,3,6,7), people on the dance floor should wave their hands in the air. And when the low bit (whose value is 1) is set (i.e. squares 1,3,5,7), people on the dance floor should shake their boody.

Give the boolean functions F_T (throw hands in air) and F_S (shake your boody) in terms of variables A, B, C, and D that equal 1 (TRUE) when the floor lights up and 0 (FALSE) otherwise. We don't care what the values of the functions are for the 8 sets of inputs that never occur. **Simplify your expressions** - people don't want to think too hard when they dance!

ANSWER:

Write the expressions for F_T and F_S:

$F_T = \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,\overline{D} + A\,B\,C\,\overline{D} + A\,B\,C\,D$

in minterm form,

$F_T = m1 + m2 + m14 + m15.$

Next, we find that the eight sets of inputs that never occur are:

m0, m3, m5, m6, m7, m9, m11, m13

Now, we can use a Karnaugh map with don't care bits to simplify the expression for F_T (see right).

$F_S = \overline{A}\,\overline{B} + B\,C$

map for F_T

A \ CD	00	01	11	10
00	X 0	1 1	X 3	1 2
01	4	X 5	X 7	X 6
11	12	X 13	1 15	1 14
10	8	X 9	X 11	10

B

Next, we do the same for F_S

$F_S = \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + A\,B\,\overline{C}\,\overline{D} + A\,B\,C\,D$

in minterm form,

$F_S = m1 + m4 + m12 + m15.$

After simplifying using a Karnaugh map, we find that

$F_T = D + B\,\overline{C}$

map for F_S

A \ CD	00	01	11	10
00	X 0	1 1	X 3	2
01	1 4	X 5	X 7	X 6
11	1 12	X 13	1 15	14
10	8	X 9	X 11	10

B