Hybrid Analog-Digital Solution of Nonlinear Partial Differential Equations

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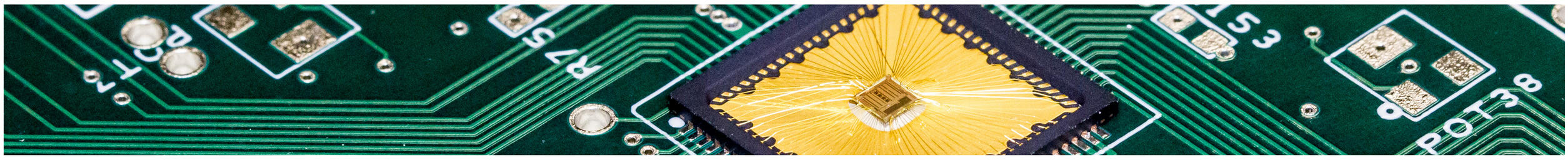
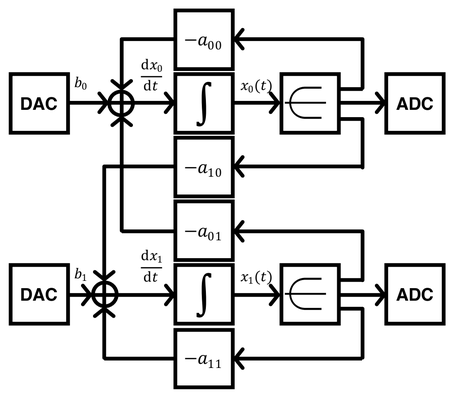
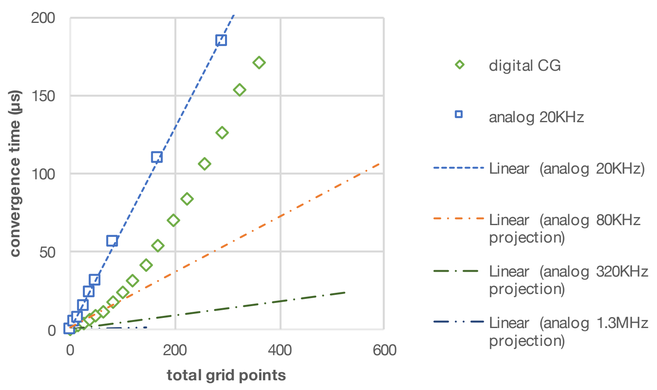


Using negative feedback analog solves linear equations; just provide constants, coefficients, and initial guesses



For smaller problem sizes & lower precision solutions, analog compares favorably against digital iterative numerical methods



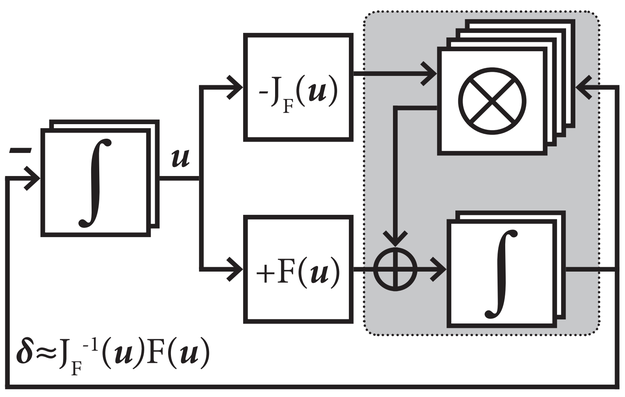
Analog linear algebra subroutine accelerates Newton's method, useful for finite difference solution of nonlinear PDEs

$$u_{\vec{p}+1} = \vec{u_p} - hJ_F^{-1}(\vec{u_p})F(\vec{u_p})$$

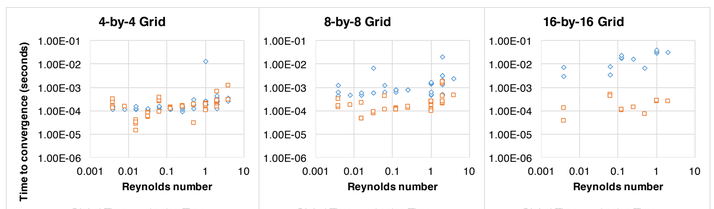
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} - \frac{1}{\mathrm{Re}}\nabla^2\vec{u} = RHS$$

$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = RHS \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = RHS \end{cases}$$

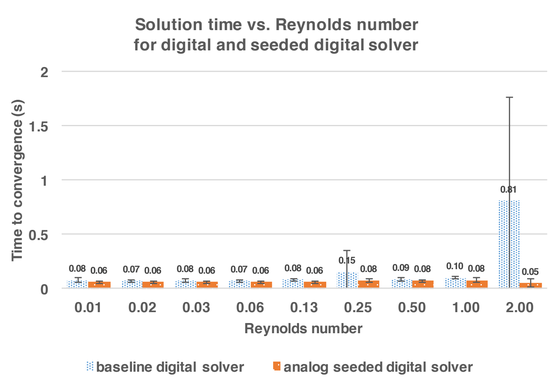
Analog continuous Newton's method takes infinitesimal steps, avoiding challenges in selecting correct initial guesses



When solving nonlinear equations resulting from Burgers' PDE, analog solves faster if problems are large and more nonlinear



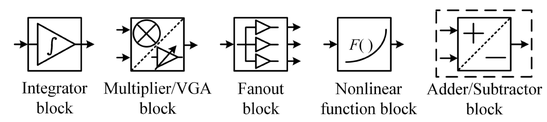
Analog approximate solutions for nonlinear equations are initial guesses for digital, which proceeds to high precision solution



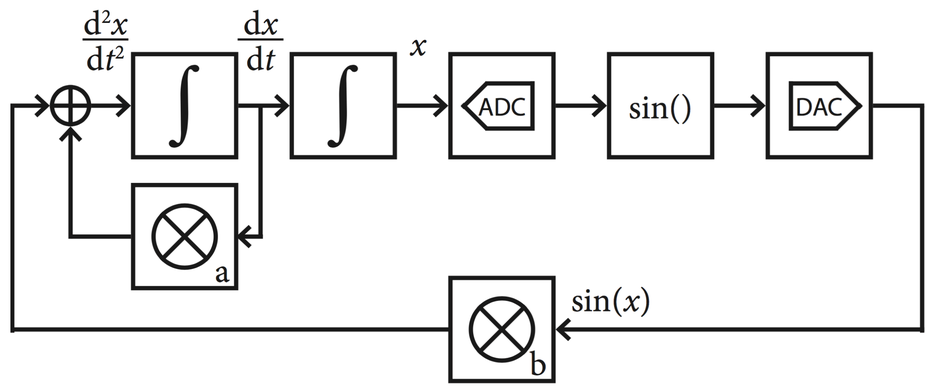
Solve nonlinear ODE using analog accelerator

$$\frac{d^2x}{dt^2} = a\frac{dx}{dt} + b\sin(x)$$

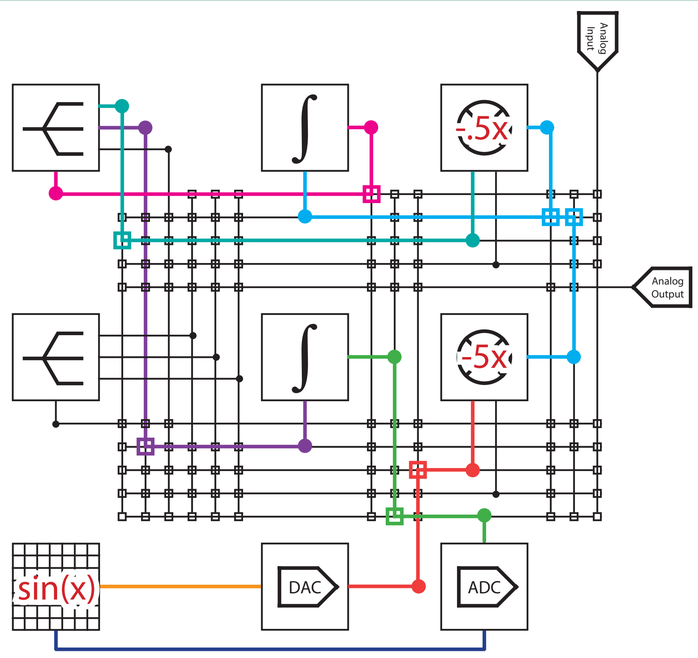
Operate on numbers encoded as voltage / current



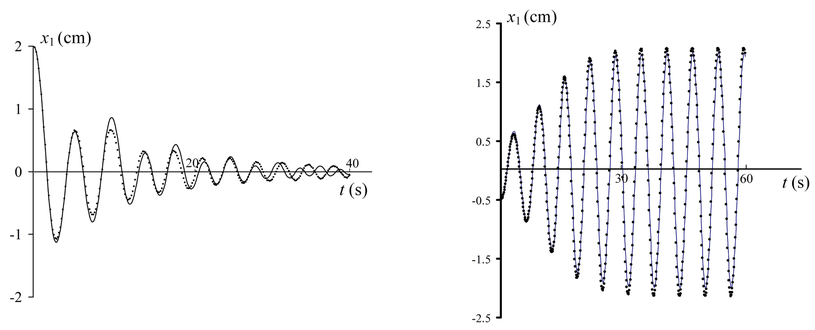
Program connections & parameters for ODE



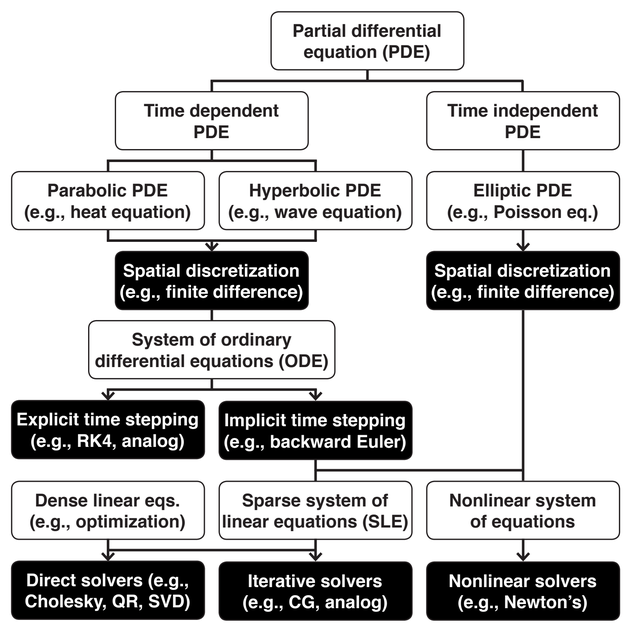
Configure circuit inside analog accelerator



Monitor / obtain analog waveform solving ODE



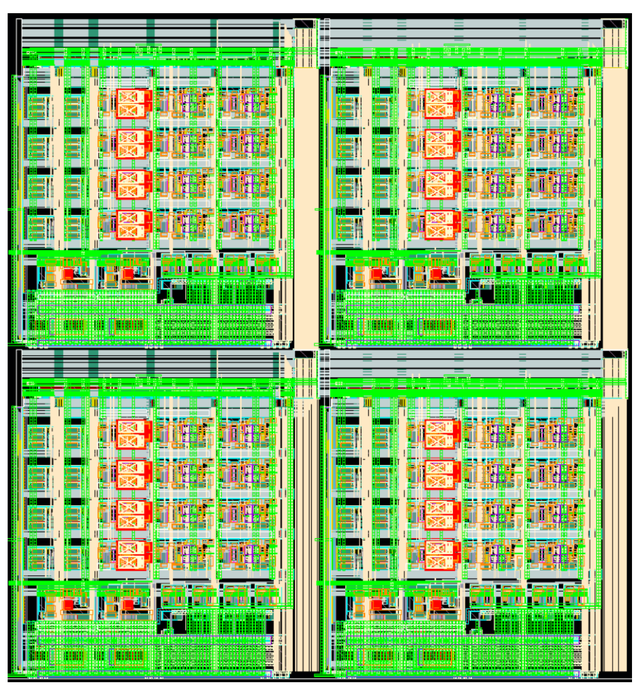
Extend ODE solving to help linear algebra & PDEs



As digital hardware improvements slow, analog holds promise in problems that are continuous, nonlinear, stochastic, approximate

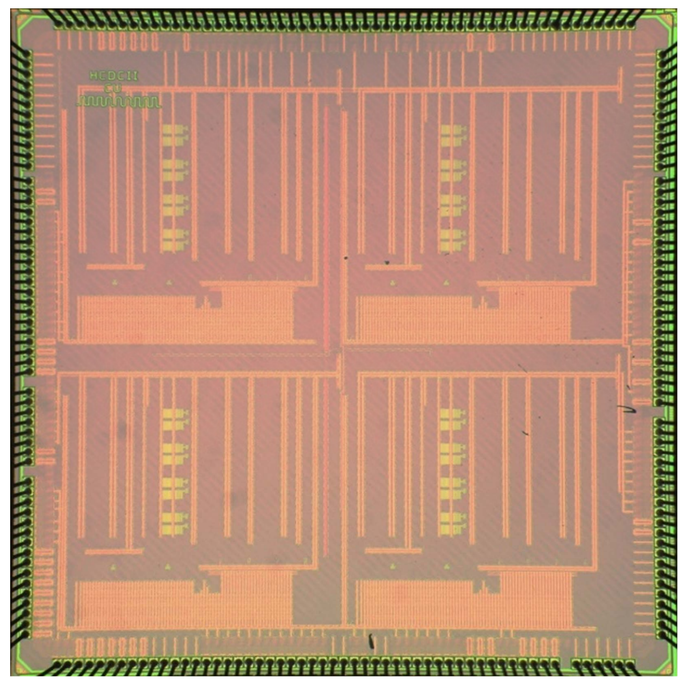
- Anticipated slowdown of speed and efficiency improvements in digital integrated circuits due to material and economic constraints
- Analog circuits deliver fast and efficient computation with existing silicon technology; analog computing history offers new directions
- Continuous-time, continuous-value hardware matches well with physical problems, which are often stochastic and nonlinear
- Downsides such as low precision and accuracy, limited scalability, and difficulty in programming can be mitigated
- Approximate solutions are useful in physical simulations and machine learning tasks for humanity's grand challenge problems

Our group's multiple generations of analog accelerator chips validate full-custom analog circuits & aid application discovery



20 KHz analog design bandwidth
16 integrators & 32 multipliers
Configurable analog crossbar
8-bit analog-digital-analog conversion
Nonlinear function lookup tables
Analog inputs & outputs for multi-chip cooperation

Physically fabricated analog accelerator chips provide ground-truth timing, area, and power consumption measurements



1.2V 65nm TSMC manufacturing process
3.7mm × 39mm dimensions
4.8 mW at full power
0.06 W/cm2 very low power density

Contact information and selected publications

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- Ning Guo, Yipeng Huang, Tao Mai, Sharvil Patil, Chi Cao, Mingoo Seok, Simha Sethumadhavan, Yannis Tsividis, "Low-Energy Hybrid Analog/Digital Approximate Computation in Continuous Time," IEEE Journal of Solid-State Circuits (JSSC), 2016.
- Yipeng Huang, Ning Guo, Mingoo Seok, Yannis Tsividis, Simha Sethumadhavan, "Evaluation of an Analog Accelerator," ACM/IEEE International Symposium on Computer Architecture (ISCA), 2016.
- Yipeng Huang, Ning Guo, Mingoo Seok, Yannis Tsividis, Kyle Mandli, Simha Sethumadhavan, "Hybrid Analog-Digital Solution of Nonlinear Partial Differential Equations," IEEE/ACM International Symposium on Microarchitecture (MICRO), 2017.

- Awarded Defense Advanced Research Projects Agency (DARPA) Small Business Technology Transfer grant to investigate modern analog computing applications.