# Quantum computing fundamentals: one qubit

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Announcements

Universal classical computing

A single qubit: the Hadamard gate, superposition, interference, measurement

### The class so far

- 1. Introductions on Canvas discussions. Important for me and classmates to know your interests.
- 2. Reading: Preskill. "Quantum Computing in the NISQ era and beyond." Describes current state of quantum computing impact and development. Discuss by posting one question and one answer—can be anything.

### Intermediate-term class plan

### Where we are headed in first month

- 1. Fundamental rules of quantum computing
- 2. Basic quantum algorithms
- 3. Programming examples in Google Cirq
- 4. A NISQ algorithm: quantum approximate optimization algorithm

5. Programming assignment on QAOA in Cirq

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# The binary abstraction

### High, low voltage Adds resilience against noise.

Representation as a state vector

• 
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$
 We pronounce this "ket" 0  
•  $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$  We pronounce this "ket" 1

# The NOT gate

Matrix representation of NOT operator:  $X = \sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ 

$$X |0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X |1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

# The SWAP gate

Matrix representation of SWAP operator: 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$SWAP |00\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle$$
$$SWAP |10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle$$

# The CNOT gate

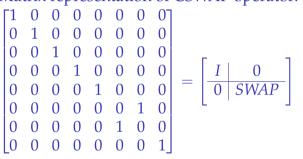
$$\begin{aligned} \text{Matrix representation of CNOT operator:} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\ \bullet & CNOT |01\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle \\ \bullet & CNOT |11\rangle &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle \end{aligned}$$

# The CSWAP gate

Matrix representation of CSWAP operator: (On the board)

# The CSWAP gate

#### Matrix representation of CSWAP operator:

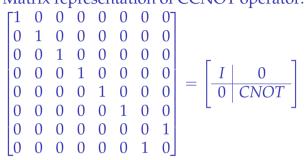


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# The CCNOT (aka Toffoli) gate

## Matrix representation of CCNOT operator:



Circuit diagram representation:

Exercise: prove that Rudolph, "Q is for Quantum," page 10, bottom figure equivalent to CCNOT

Universal classical computation

# Toffoli (CCNOT) gate can represent all classical computation (How?)

# Universal classical computation

### Toffoli (CCNOT) gate can represent all classical computation

1. All Boolean expressions can be phrased as either CNF or DNF.

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- 2. AND, OR, and NOT operations are universal.
- 3. Either NAND or NOR are individually universal.
- 4. CCNOT implements NAND. (Feed  $|1\rangle$  into target qubit).
- 5. So, CCNOT is universal for classical logic.

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### The Hadamard gate

Matrix representation of Hadamard operator:  $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ 

$$H |0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$
$$H |1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$$

# Superposition

### Single qubit state

- $\blacktriangleright \ \alpha \left| 0 \right\rangle + \beta \left| 1 \right\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$
- Amplitudes  $\alpha, \beta \in \mathbb{C}$
- $\blacktriangleright \ |\alpha|^2 + |\beta|^2 = 1$
- ► The above constraints require that qubit operators are unitary matrices.

# Many physical phenomena can be in superposition and encode qubits

- Polarization of light in different directions
- Electron spins (Intel solid state qubits)
- Atom energy states (UMD, IonQ ion trap qubits)
- Quantized voltage and current (IBM, Google superconducting qubits)

If multiple discrete values are possible (e.g., atom energy states, voltage and current), we pick (bottom) two for the binary abstraction.

### Interference

### Amplitudes can positively and negatively interfere

$$HH |0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$HH |1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

### Measurement

These rules, states, and operators model real quantum phenomena

States in our examples cannot be merely classical or probabilistic

### Double slit experiment

https://www.youtube.com/watch?v=Q1YqgPAtzho

### Textbook formalism

For an introductory textbook on the quantum computing formalism, I recommend:

https://www.lassp.cornell.edu/mermin/qcomp/CS483.html

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