Quantum computing fundamentals: dense coding, Bell's inequality

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Announcements

First programming exercise

https://rutgers.instructure.com/courses/140409/assignments/1660355?module_item_id=5387614

Intermediate-term class plan

Where we are headed in first month

1. Fundamentals: superposition / Deutsch-Jozsa
2. Fundamentals: entanglement / Bell inequalities
3. Programming examples in Google Cirq
4. Shor's algorithm (new)
5. A NISQ algorithm: quantum approximate optimization algorithm
6. Programming assignment on QAOA in Cirq

Longer-term class plan

The remaining two months

1. Systems view of quantum computing
2. Programming abstractions: stabilizers, tensor networks
3. VQE: quantum chemistry
4. Quantum error correction codes (new)
5. Quantum architecture and micro-architecture
6. Prototype devices: superconductors and ion traps

What is being cut

1. 2020 quantum advantage debates
2. Student presentations: $2 \to 1$

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The lens we take when studying quantum computing

- When quantum physics was first discovered, the mathematics of entanglement led to shocking conclusions.
- If you can keep systems coherent (isolated), they can exhibit superposition and entanglement.
- Einstein and others: there shouldn't be "spooky action at a distance" so there must be some local hidden-variable. The task was then to prove or disprove local hidden-variables.
- But protocols and experiments like Hardy's, GHZ, CHSH, and Aspect experimentally rejected local hidden-variable theory.

The lens we take when studying quantum computing

Cannot have both locality and realism

- Locality: "means that information and causation act locally, not faster than light"
- Realism: "means that physical systems have definite, well-defined properties (even if those properties may be unknown to us)"

Source: de Wolf. Quantum Computing: Lecture Notes

Unpalatable choices

- Keep locality and sacrifice realism: no definite narrative of the world
- Keep realism and sacrifice locality: spooky-action-at-a-distance

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Prove that the Bell state cannot be factored into two single-qubit states

Bell state circuit

$$|00\rangle \xrightarrow{H\otimes I} \frac{1}{\sqrt{2}}\Big(|00\rangle + |10\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle \Big) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |\Phi^+\rangle$$

Can $|\Phi^+\rangle$ be treated as the tensor product (composition) of two individual qubits?
No.

Proof by contradiction:

- Suppose $|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \Big(\alpha|0\rangle + \beta|1\rangle\Big) \otimes \Big(\gamma|0\rangle + \delta|1\rangle\Big) = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$.
- $\alpha\delta = 0$, so either $\alpha = 0$ or $\delta = 0$.
- But $\alpha\gamma = \frac{1}{\sqrt{2}}$, so $\alpha \neq 0$.
- And $\beta\delta = \frac{1}{\sqrt{2}}$, so $\delta \neq 0$ too.

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Quantum teleportation

"Teleport" a qubit state by transmitting classical information

1. Alice wishes to give Bob a qubit state $|Q\rangle$.
2. Alice and Bob each have one qubit of a Bell pair in state $|P\rangle$.
3. Alice first entangles $|Q\rangle$ and $|P\rangle$; then, she measures her local two qubits.
4. Alice tells Bob (via classical means) her two-bit measurement result.
5. Bob uses Alice's two bits to perform I, X, Z, or ZX on his qubit to obtain $|Q\rangle$.

Depending on if Alice measures 00, 01, 10, or 11, Bob applies I, X, Z, or ZX to recover $|Q\rangle$

$$
\begin{aligned}
|Q\rangle \otimes |P\rangle &= \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \\
&= \frac{\alpha}{\sqrt{2}}|0\rangle\left(|00\rangle + |11\rangle\right) + \frac{\beta}{\sqrt{2}}|1\rangle\left(|00\rangle + |11\rangle\right) \\
&\xrightarrow{CNOT_{0,1}} \frac{\alpha}{\sqrt{2}}|0\rangle\left(|00\rangle + |11\rangle\right) + \frac{\beta}{\sqrt{2}}|1\rangle\left(|10\rangle + |01\rangle\right) \\
&\xrightarrow{H\otimes I\otimes I} \frac{\alpha}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\left(|00\rangle + |11\rangle\right) + \frac{\beta}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\left(|10\rangle + |01\rangle\right) \\
&= \frac{1}{2}|00\rangle\left(\alpha|0\rangle + \beta|1\rangle\right) \quad \text{Alice measures 00, Bob applies I} \\
&+ \frac{1}{2}|01\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) \quad \text{Alice measures 01, Bob applies X} \\
&+ \frac{1}{2}|10\rangle\left(\alpha|0\rangle - \beta|1\rangle\right) \quad \text{Alice measures 10, Bob applies Z} \\
&+ \frac{1}{2}|11\rangle\left(\alpha|1\rangle - \beta|0\rangle\right) \quad \text{Alice measures 11, Bob applies ZX}
\end{aligned}
$$

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Superdense coding

Transmit 2 bits of classical information by sending 1 qubit

1. Alice wishes to tell Bob two bits of information: 00, 01, 10, or 11.
2. Alice and Bob each have one qubit of a Bell pair in state $|P\rangle = |\Phi^+\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)$.
3. Alice performs I, X, Z, or ZX on her qubit; she then sends her qubit to Bob.
4. Bob measures in the Bell basis to receive 00, 01, 10, or 11.

Alice applies different operators on her qubit so Bob measures the message

1. $|P\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big) \xrightarrow{I\otimes I} \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}\Big(|00\rangle + |10\rangle\Big) \xrightarrow{H\otimes I} |00\rangle$
2. $|P\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big) \xrightarrow{X\otimes I} \frac{1}{\sqrt{2}}\Big(|10\rangle + |01\rangle\Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}\Big(|11\rangle + |01\rangle\Big) \xrightarrow{H\otimes I} |01\rangle$
3. $|P\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big) \xrightarrow{Z\otimes I} \frac{1}{\sqrt{2}}\Big(|00\rangle - |11\rangle\Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}\Big(|00\rangle - |10\rangle\Big) \xrightarrow{H\otimes I} |10\rangle$
4. $|P\rangle = \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big) \xrightarrow{ZX\otimes I} \frac{1}{\sqrt{2}}\Big(-|10\rangle + |01\rangle\Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}\Big(-|11\rangle + |01\rangle\Big) \xrightarrow{H\otimes I} |11\rangle$

Channel capacity of dense coding

How does dense coding scale?

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Dense coding with multipartite qubit entanglement

3 qubit maximally entangled orthogonal basis set

1. $|000\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|000\rangle + |100\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big)$
2. $|001\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|001\rangle + |101\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|001\rangle + |110\rangle\Big)$
3. $|010\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|010\rangle + |110\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|010\rangle + |101\rangle\Big)$
4. $|011\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|011\rangle + |111\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|011\rangle + |100\rangle\Big)$
5. $|100\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|000\rangle - |100\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|000\rangle - |111\rangle\Big)$
6. $|101\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|001\rangle - |101\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|001\rangle - |110\rangle\Big)$
7. $|110\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|010\rangle - |110\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|010\rangle - |101\rangle\Big)$
8. $|111\rangle \xrightarrow{H\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|011\rangle - |111\rangle\Big) \xrightarrow{CNOT_{0,1}, CNOT_{0,2}} \frac{1}{\sqrt{2}}\Big(|011\rangle - |100\rangle\Big)$

Exercise: show that this is an orthogonal set of vectors.

Transmit 3 bits of classical information by sending 2 qubits

Alice applies different operators on her qubits so Bob measures the message

1. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{I\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|000\rangle + |100\rangle\Big) \xrightarrow{H\otimes I\otimes I} |000\rangle$
2. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{X\otimes X\otimes I} \frac{1}{\sqrt{2}}\Big(|110\rangle + |001\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|101\rangle + |001\rangle\Big) \xrightarrow{H\otimes I\otimes I} |001\rangle$
3. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{I\otimes X\otimes I} \frac{1}{\sqrt{2}}\Big(|010\rangle + |101\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|010\rangle + |110\rangle\Big) \xrightarrow{H\otimes I\otimes I} |010\rangle$
4. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{X\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|100\rangle + |011\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|111\rangle + |011\rangle\Big) \xrightarrow{H\otimes I\otimes I} |011\rangle$
5. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{Z\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(|000\rangle - |111\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|000\rangle - |100\rangle\Big) \xrightarrow{H\otimes I\otimes I} |100\rangle$
6. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{ZX\otimes X\otimes I} \frac{1}{\sqrt{2}}\Big(-|110\rangle + |001\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(-|101\rangle + |001\rangle\Big) \xrightarrow{H\otimes I\otimes I} |101\rangle$
7. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{Z\otimes X\otimes I} \frac{1}{\sqrt{2}}\Big(|010\rangle - |101\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(|010\rangle - |110\rangle\Big) \xrightarrow{H\otimes I\otimes I} |110\rangle$
8. $\frac{1}{\sqrt{2}}\Big(|000\rangle + |111\rangle\Big) \xrightarrow{ZX\otimes I\otimes I} \frac{1}{\sqrt{2}}\Big(-|100\rangle + |011\rangle\Big) \xrightarrow{CNOT_{0,2},CNOT_{0,1}} \frac{1}{\sqrt{2}}\Big(-|111\rangle + |011\rangle\Big) \xrightarrow{H\otimes I\otimes I} |111\rangle$

Transmit 4 bits of classical information by sending 2 qubits

4 qubit maximally entangled orthogonal basis set

Gustavo Rigolin. Superdense coding using multipartite states. arXiv. 2004.
`https://arxiv.org/pdf/quant-ph/0407193.pdf`

Dense coding with multipartite qubit entanglement

Maximum dense coding channel capacity of 2 bits received per *qubit* sent.

Can one do better if using something other than *qubits*?

Hu et al.. Beating the channel capacity limit for superdense coding with entangled ququarts. Science Advances. 2018.
`https://www.science.org/doi/10.1126/sciadv.aat9304`

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Dense coding with high-dimensional qudit entanglement

Qudits

	1 qudit basis vectors	2 qudit basis vectors	3 qudit basis vectors
Qubit	$\|0\rangle, \|1\rangle$	$\|00\rangle, \|01\rangle, \|10\rangle, \|11\rangle$	$\|000\rangle, \|001\rangle, \|010\rangle, ..., \|111\rangle$
Qutrit	$\|0\rangle, \|1\rangle, \|2\rangle$	$\|00\rangle, \|01\rangle, \|02\rangle, \|10\rangle, ..., \|22\rangle$	$\|000\rangle, ..., \|002\rangle, \|010\rangle, ..., \|222\rangle$
Ququart	$\|0\rangle, \|1\rangle, \|2\rangle, \|3\rangle$	$\|00\rangle, ..., \|03\rangle, \|10\rangle, ..., \|33\rangle$	$\|000\rangle, ..., \|003\rangle, \|010\rangle, ..., \|333\rangle$

So for example, 2 qutrits with an orthogonal state space size of $3^2 = 9$ can encode the state space of 3 qubits (which has state space size of $2^3 = 8$).

DFT matrix: generalized Hadamard for D-dimensional qudits

$$W = \frac{1}{\sqrt{D}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{D-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(D-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{D-1} & \omega^{2(D-1)} & \cdots & \omega^{(D-1)(D-1)} \end{bmatrix}$$

Where

$$\omega = e^{\frac{2}{D}\pi i}$$

And recall that

$$e^{ix} = \cos x + i \sin x$$

Teo Banica. Complex Hadamard matrices and applications. 2021. hal-02317067v2
`https://arxiv.org/abs/1910.06911`

DFT matrix: generalized Hadamard for D-dimensional qudits

$D = 2$, Hadamard matrix for qubits

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & \omega^1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & e^{\frac{2}{2}\pi i} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$D = 3$, Hadamard matrix for qutrits

$$W = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2}{3}\pi i} & e^{\frac{4}{3}\pi i} \\ 1 & e^{\frac{4}{3}\pi i} & e^{\frac{8}{3}\pi i} \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2}{3}\pi i} & e^{\frac{-2}{3}\pi i} \\ 1 & e^{\frac{-2}{3}\pi i} & e^{\frac{2}{3}\pi i} \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}}{2}i & -\frac{1}{2} - \frac{\sqrt{3}}{2}i \\ 1 & -\frac{1}{2} - \frac{\sqrt{3}}{2}i & -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{bmatrix}$$

Yurtalan et al.. Implementation of a Walsh-Hadamard Gate in a Superconducting Qutrit. PRL. 2020.
https://journals.aps.org/prl/

Generalized CNOT gate

$D = 2$, CNOT for qubits

$|x, y\rangle \xrightarrow{CNOT} |x, x + y \mod 2\rangle$

1. $|0, 0\rangle \xrightarrow{CNOT} |0, 0\rangle$
2. $|0, 1\rangle \xrightarrow{CNOT} |0, 1\rangle$
3. $|1, 0\rangle \xrightarrow{CNOT} |1, 1\rangle$
4. $|1, 1\rangle \xrightarrow{CNOT} |1, 0\rangle$

$D = 3$, CPLUS for qutrits

$|x, y\rangle \xrightarrow{CPLUS} |x, x + y \mod 3\rangle$

1. $|0, 0\rangle \xrightarrow{CPLUS} |0, 0\rangle$
2. $|0, 1\rangle \xrightarrow{CPLUS} |0, 1\rangle$
3. $|0, 2\rangle \xrightarrow{CPLUS} |0, 2\rangle$
4. $|1, 0\rangle \xrightarrow{CPLUS} |1, 1\rangle$
5. $|1, 1\rangle \xrightarrow{CPLUS} |1, 2\rangle$
6. $|1, 2\rangle \xrightarrow{CPLUS} |1, 0\rangle$
7. $|2, 0\rangle \xrightarrow{CPLUS} |2, 2\rangle$
8. $|2, 1\rangle \xrightarrow{CPLUS} |2, 0\rangle$
9. $|2, 2\rangle \xrightarrow{CPLUS} |2, 1\rangle$

Çorbaci et al.. Construction of two qutrit entanglement by using magnetic resonance selective pulse sequences. Journal of Physics: Conference Series. 2016. `https://iopscience.iop.org/article/10.1088/1742-6596/766/1/012014`

Dense coding with high-dimensional qudit entanglement

2 qutrit maximally entangled orthogonal basis set

1. $|00\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|00\rangle + |10\rangle + |20\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|00\rangle + |11\rangle + |22\rangle\Big)$
2. $|01\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|01\rangle + |11\rangle + |21\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|01\rangle + |12\rangle + |20\rangle\Big)$
3. $|02\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|02\rangle + |12\rangle + |22\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|02\rangle + |10\rangle + |21\rangle\Big)$
4. $|10\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|00\rangle + \omega\,|10\rangle + \omega^2\,|20\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|00\rangle + \omega\,|11\rangle + \omega^2\,|22\rangle\Big)$
5. $|11\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|01\rangle + \omega\,|11\rangle + \omega^2\,|21\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|01\rangle + \omega\,|12\rangle + \omega^2\,|20\rangle\Big)$
6. $|12\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|02\rangle + \omega\,|12\rangle + \omega^2\,|22\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|02\rangle + \omega\,|10\rangle + \omega^2\,|21\rangle\Big)$
7. $|20\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|00\rangle + \omega^2\,|10\rangle + \omega\,|20\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|00\rangle + \omega^2\,|11\rangle + \omega\,|22\rangle\Big)$
8. $|21\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|01\rangle + \omega^2\,|11\rangle + \omega\,|21\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|01\rangle + \omega^2\,|12\rangle + \omega\,|20\rangle\Big)$
9. $|22\rangle \xrightarrow{W\otimes I} \frac{1}{\sqrt{3}}\Big(|02\rangle + \omega^2\,|12\rangle + \omega\,|22\rangle\Big) \xrightarrow{CPLUS} \frac{1}{\sqrt{3}}\Big(|02\rangle + \omega^2\,|10\rangle + \omega\,|21\rangle\Big)$

Exercise: show that this is an orthogonal set of vectors.

Focus on lines 4 and 7. Recall complex vector dot product.

Breaking the binary abstraction

Is ternary (and beyond) logic useful in classical computing?

- Historically was useful in analog computing.
- Drive to decrease supply voltage while increasing noise resilience made processing in binary the dominant paradigm.
- Current still has uses in storage, at least.

Is ternary (and beyond) logic useful in quantum computing?

- PRO: higher quantum channel capacity.
- CON: qubits are much easier to engineer that qutrits.

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Dense coding programs and systems

In this class, we study quantum algorithms down to hardware

- https://quantumai.google/cirq/qudits
- https://qiskit.org/textbook/ch-quantum-hardware/accessing_higher_energy_states.html
- Cervera-Lierta. Experimental high-dimensional Greenberger-Horne-Zeilinger entanglement with superconducting transmon qutrits. arXiv. 2021. https://arxiv.org/abs/2104.05627

Challenge programming assignment: qutrit dense coding in Cirq

Can substitute either programming assignment on QAOA or VQE.

Timely research project: world-first demo of qutrit dense coding on IBM Q

Likely key challenge: qubits currently engineered for CNOTs.