Graphical models and logical abstractions for quantum systems

Yipeng Huang

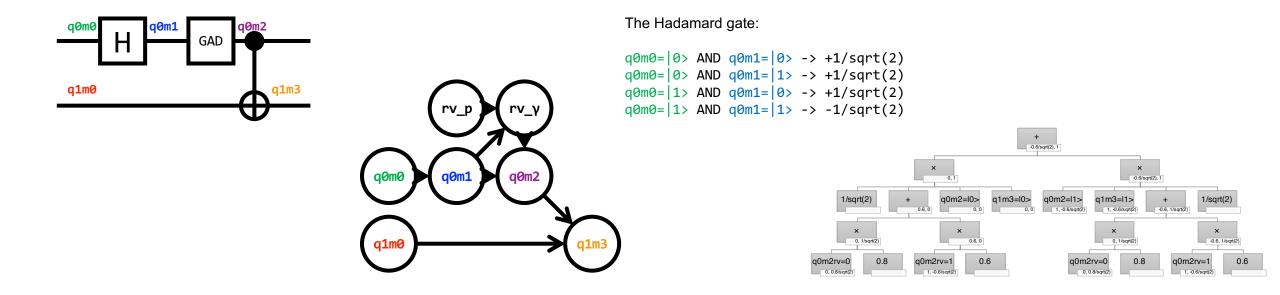
November 15, 2021



What this talk is about:

Using classical probabilistic inference techniques as an abstraction for quantum computing.

- A new way to represent noisy quantum circuits as probabilistic graphical models.
- A new way to encode quantum circuits as conjunctive normal forms and arithmetic circuits.
- A new way to manipulate quantum circuits using logical equation satisfiability solvers.
- Improved simulation and sampling performance for important near-term quantum algorithms.



Where we are going:

What are quantum variational algorithms?

Why are they different and important?

What is quantum circuit simulation?

Why are the conventional techniques insufficient?

How do we represent quantum circuits as logic formulas?

Why does it help with variational algorithm simulation, and by how much?

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Limitations of near-term quantum computers

- Limited number of qubits (the fundamental information units and devices in quantum computing).
- Noisy, unreliable operations.
- Limited operations on each qubit.
- Error correction too costly (needs ~million qubits), not available.

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NISQ systems target variational algorithms.

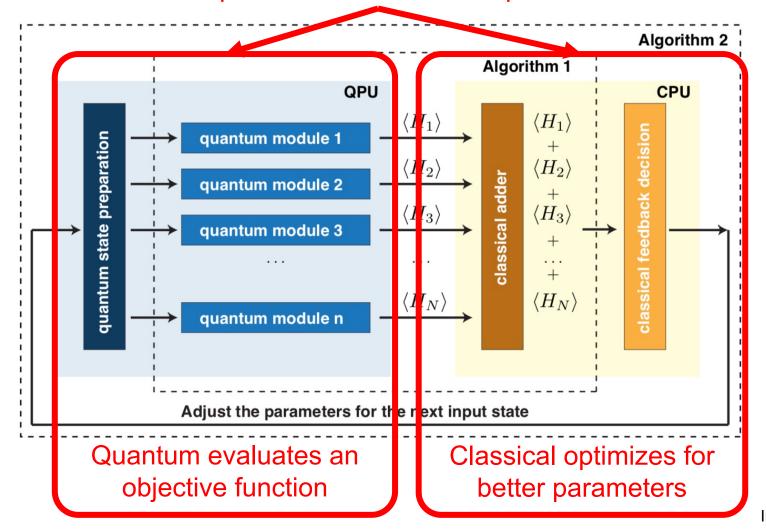
Near-term Intermediate Scale Quantum (NISQ) systems have ~100 qubits with at best 0.1% error rate.

With that capacity and reliability, error correction, along with famous algorithms such as Grover's search and Shor's factoring are infeasible.

The soonest candidates for useful quantum computation involve quantum-classical variational algorithms.

Hybrid quantum-classical variational algos

Use quantum & classical computation



It's like using a classical computer to train a quantum neural network.

Image source: Peruzzo et al., 2013

Unique traits of variational algorithms

Provides meaningful results with noise even without error correction.

Draws on strengths of quantum and classical:

- Repeatedly prepare and measure quantum states.
- Optimize for a set of optimal parameters based on classical measurements.

Wide but shallow circuits (not many operations on many qubits).

Specific examples of variational algorithms

Variational quantum eigensolver (VQE)

Simulate quantum mechanics.

Quantum approximate optimization algorithm (QAOA)

Approximate solutions to constraint satisfaction problems (CSPs).

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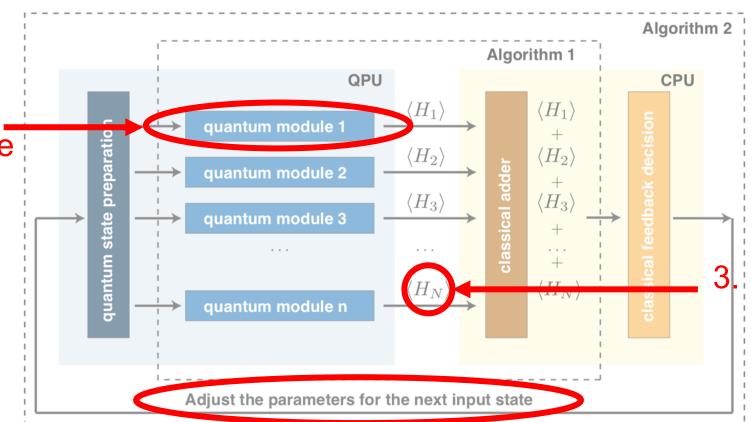
Why are the conventional techniques insufficient?

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The unique challenge of simulating noisy variational algorithms

1. Needs to simulate noise (independent and correlated)



Only need samples, not full wavefunctions.

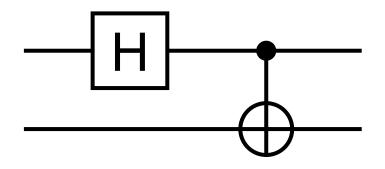
2. Require repeated simulation with different parameters

Rock, paper, scissors: Existing simulation techniques are not suited for variational algorithms

Schrödinger simulation

QuEST, IBM, Google; parallel matrix vector multiplication

Schrödinger quantum circuit simulation



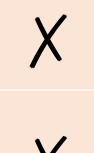
$$\mathsf{CNOT}(\mathsf{H} \otimes \mathsf{I}|00\rangle) = \mathsf{CNOT}(\mathsf{H}|0\rangle \otimes \mathsf{I}|0\rangle) = \mathsf{CNOT}\begin{bmatrix}\frac{1}{\sqrt{2}} \begin{bmatrix}1\\0\end{bmatrix} \\ \frac{1}{\sqrt{2}} \begin{bmatrix}1\\0\end{bmatrix} \end{bmatrix} = \begin{bmatrix}1\\&1\\&&1\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Rock, paper, scissors: Existing simulation techniques are not suited for variational algorithms

Schrödinger simulation

QuEST, qSim, ...; parallel matrix vector multiplication

- 1. Does it excel at simulating wide but shallow circuits?
- 2. Does it extract structure for repeated simulation with different parameters?
- 3. Does it efficiently sample from the final wavefunction?







Rock, paper, scissors: Existing simulation techniques are not suited for variational algorithms

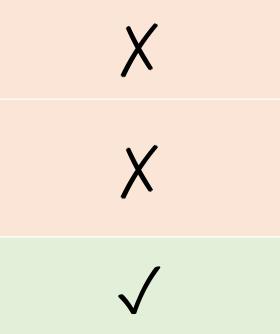
Schrödinger simulation

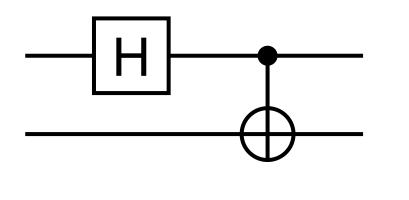
QuEST, qSim, ...; parallel matrix vector multiplication

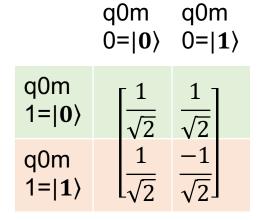
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Feynman simulation

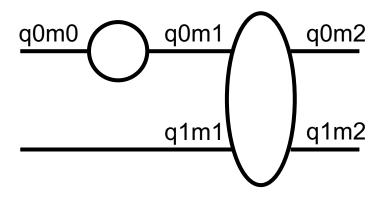
qTorch; graphical model tensor network contraction







Feynman quantum circuit simulation



q0m2= 0 >	q1m2= 0
	q1m2= 1
q0m2= 1 >	q1m2= 0
	q1m2= 1

q0m1= 0 >		q0m1= 1 >	
q1m1= 0 >	q1m1= 1 >	q1m1= 0 >	q1m1= 1 >
1	0	0	0
0	1	0	0
0	0	0	1
0	0	1	0

q0m0		q0m2
	q1m1	<u>q1m2</u>

q0m2= 0 >	q1m2= 0 >
	q1m2= 1 >
q0m2= 1 >	q1m2= 0 >
	q1m2= 1 >

q0m0	$0= 0\rangle$	q0m(0= 1>
$q1m1= 0\rangle$	q1m1= 1 >	q1m1= 0 >	$q1m1= 1\rangle$
$1/\sqrt{2}$	0	$1/\sqrt{2}$	0
0	$1/\sqrt{2}$	0	$1/\sqrt{2}$
0	$1/\sqrt{2}$	0	$-1/\sqrt{2}$
$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0

Rock, paper, scissors: Existing simulation techniques are not suited for variational algorithms

Schrödinger simulation	Feynman simulation	
QuEST, IBM, Google; parallel matrix vector multiplication	qTorch; graphical model tensor network contraction	
X		
X	?	
	X	

1. Does it excel at

simulating wide but

shallow circuits?

2. Does it extract

parameters?

wavefunction?

structure for repeated

3. Does it efficiently

sample from the final

simulation with different

Where we are going:

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Why are the conventional techniques insufficient?

How do we represent quantum circuits as logic formulas?

Why does it help with variational algorithm simulation, and by how much?

- 1. Noisy quantum circuits to Bayesian network
- 2. Bayesian networks to conjunctive normal form (CNF)
- 3. CNF to arithmetic circuit (AC)
- 4. Exact inference on AC for quantum circuit simulation
- 5. Gibbs sampling on AC to sample from final wavefunction

1. Noisy quantum circuits to Bayesian network

 Needs to simulate noise

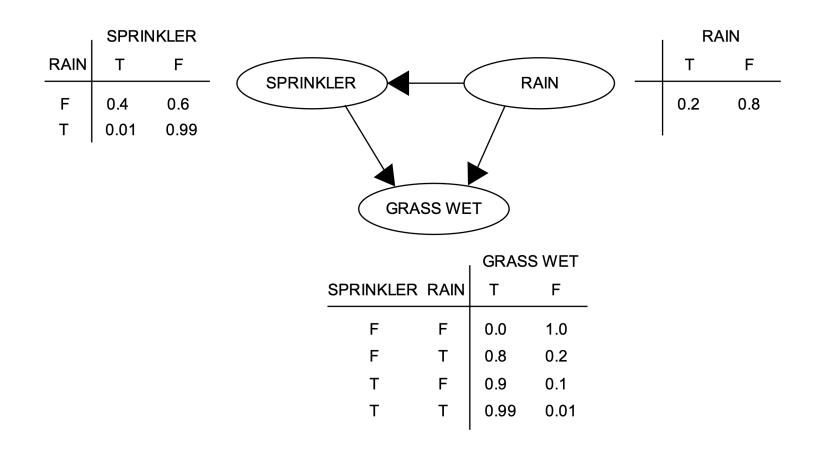
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2. Repeated simulation with different parameters

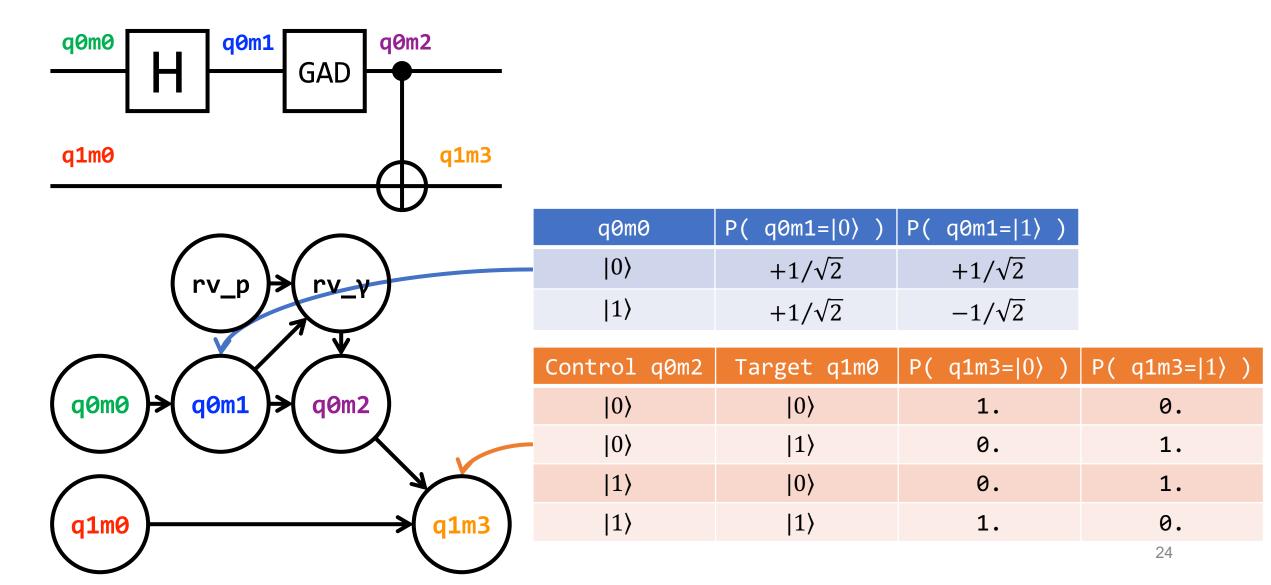
- 5. Gibbs sampling on AC to sample from final wavefunction
- Only need samples, not full wavefunctions

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Bayesian networks: AI models that encode probabilistic knowledge in a factorized format



Noisy quantum circuits to Bayesian network



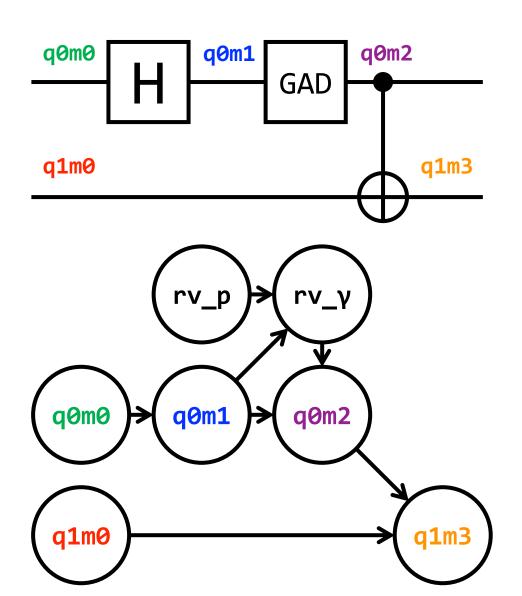
Connection between quantum circuits and probabilistic graphical models

	Quantum	Probabilistic
Key analogies	program simulation qubits amplitudes operator unitary matrices superposition states entangled qubits measurement	inference random variables probabilities conditional probability tables probability distributions dependent random variables sampling & conditioning
Key distinctions	amplitudes are complex-valued squares of amplitudes sum to 1 interference (canceling of amplitudes) possible	probabilities between 0 and 1 probabilities sum to 1 interference impossible

Quantum / probabilistic:

Separated by Gottesman-Knill theorem, ideas can cross-pollinate

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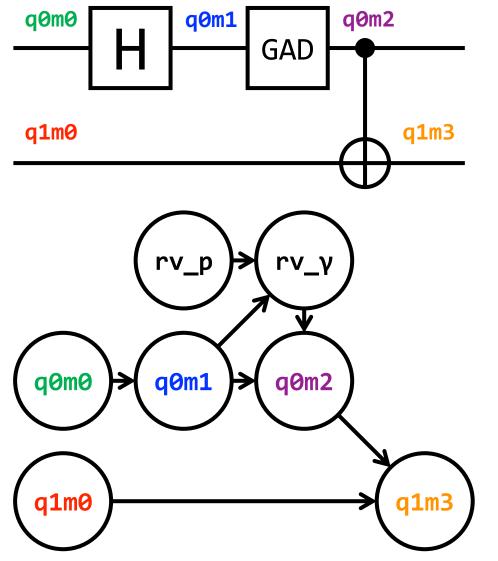


Think about circuit as logic equation

Compile & minimize this logic equation

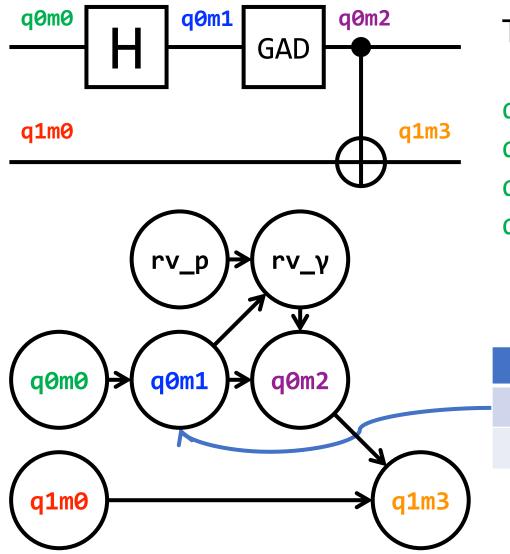
Variable assignments that satisfy CNF are valid Feynman paths through algorithm

 Model count on variable assignments yields quantum circuit simulation



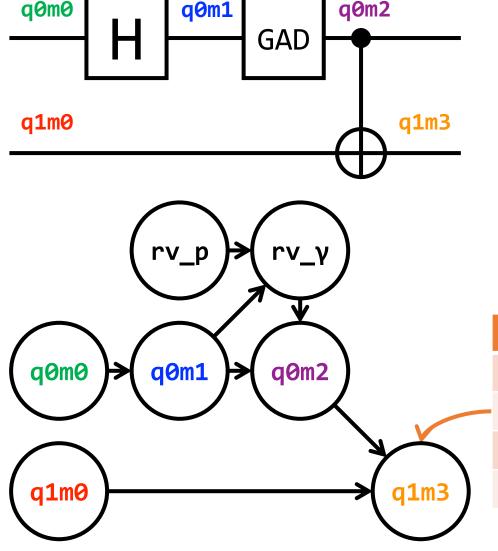
Qubits take on binary values:

```
q0m0= | 0 > XOR q0m0= | 1 >
q0m1= | 0 > XOR q0m1= | 1 >
q0m2= | 0 > XOR q0m2= | 1 >
q1m0= | 0 > XOR q1m0= | 1 >
q1m3= | 0 > XOR q1m3= | 1 >
```



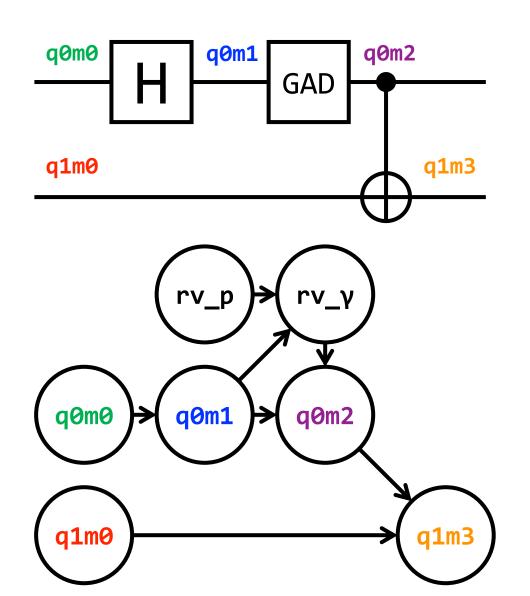
The Hadamard gate:

q0m0	P($q@m1= 0\rangle$)	P(q0m1= 1>)
0>	$+1/\sqrt{2}$	$+1/\sqrt{2}$
1>	$+1/\sqrt{2}$	$-1/\sqrt{2}$



The CNOT gate:

Control q0m2	Target q1m0	P(q1m3= $ 0\rangle$)	P(q1m3= $ 1\rangle$)
0>	0>	1.	0.
0>	1>	0.	1.
1>	0>	0.	1.
1>	1>	1.	0.



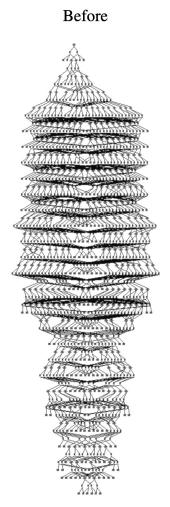
Put all the sentences together!

Convert logical implications "→" to logical disjunctions

Conjoin all the disjunctive clauses together to form CNF (i.e., AND all the ORs together)

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CNF to arithmetic circuit (AC)



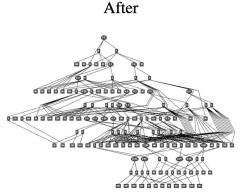


Figure 1: Equivalent knowledge compilation representations of a 4qubit noisy QAOA quantum circuit. In this work we calculate and sample amplitudes from arithmetic circuits (ACs) representing noisy quantum circuits. To the left, direct compilation results in ACs where qubit states ordered in time increases from top to bottom. Above, logical minimization, variable reordering, and eliding internal states reduces the size of the AC. The equivalent reduced representation leads to more efficient simulation and sampling.

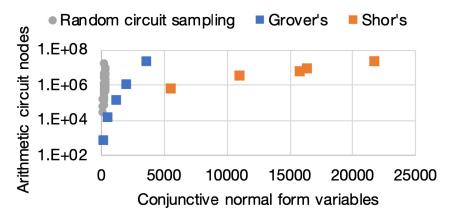
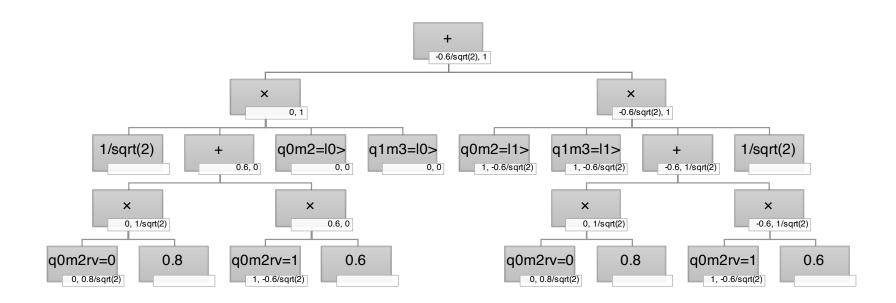


Figure 6: Simulation resource requirements vs. quantum circuit size for three quantum algorithms

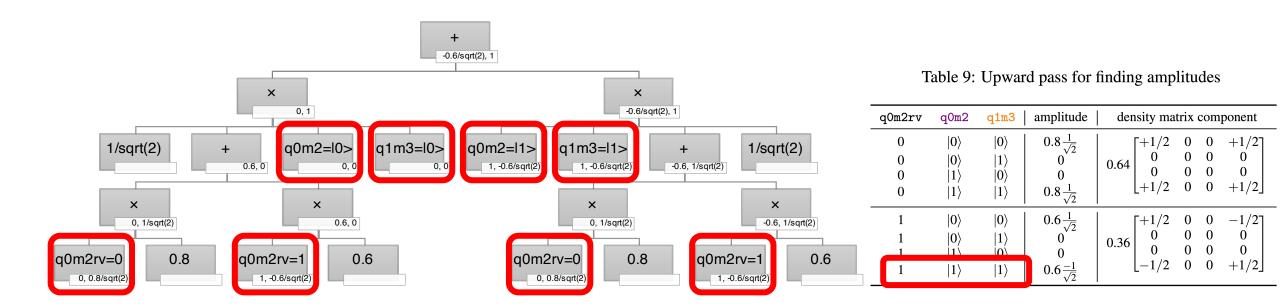
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Exact inference on AC for quantum circuit simulation



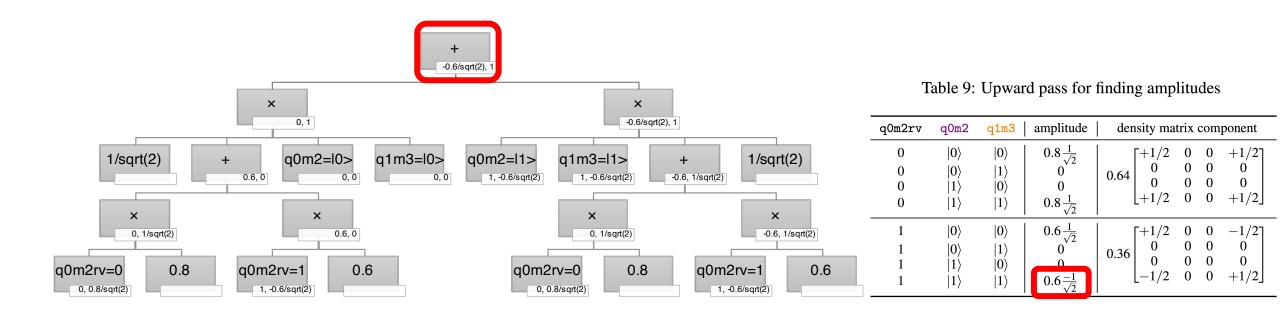
Quantum simulation becomes tree traversal on AC

Exact inference on AC for quantum circuit simulation



- Quantum simulation becomes tree traversal on AC
- Quantum measurement outcomes are probabilistic evidence

Exact inference on AC for quantum circuit simulation



- Quantum simulation becomes tree traversal on AC
- Quantum measurement outcomes are probabilistic evidence
- Amplitude for given outcome comes from root node

Our toolchain: Bayesian network knowledge compilation for noisy quantum circuit simulation and sampling

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Gibbs sampling on AC to sample from final wavefunction

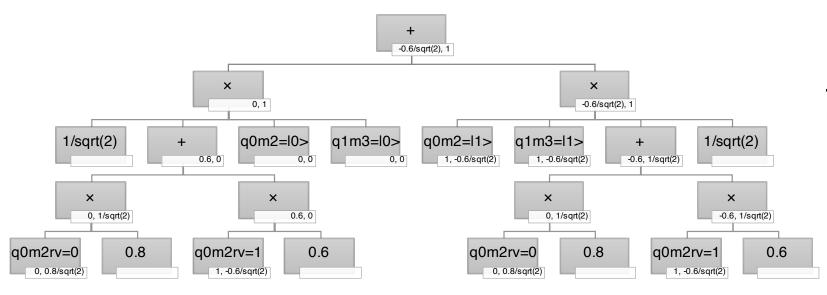
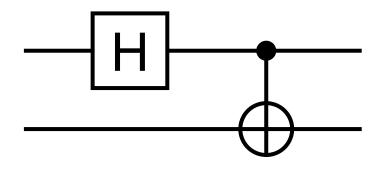


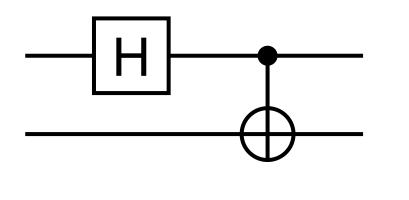
Table 10: Downward pass for finding derivatives for Gibbs sampling MCMC

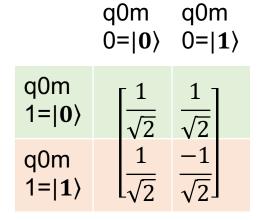
	q0m2rv	q0m2	q1m3 amplitude
Present sample	1	$ 1\rangle$	$ 1\rangle$ $0.6\frac{-1}{\sqrt{2}}$
Gibbs sample noise	0	$ 1\rangle$	$ 1\rangle 0.8\frac{1}{\sqrt{2}}$
Gibbs sample qubits	1	$ 0\rangle$	$ 1\rangle$ $0^{\sqrt{2}}$
Gibbs sample qubits	1	$ 1\rangle$	$ 0\rangle$ 0

Schrödinger quantum circuit simulation

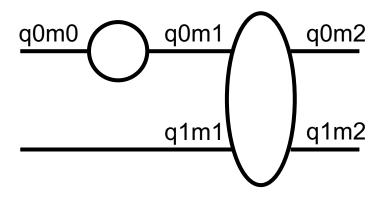


$$\mathsf{CNOT}(\mathsf{H} \otimes \mathsf{I}|00\rangle) = \mathsf{CNOT}(\mathsf{H}|0\rangle \otimes \mathsf{I}|0\rangle) = \mathsf{CNOT}\begin{bmatrix}\frac{1}{\sqrt{2}} \begin{bmatrix}1\\0\end{bmatrix}\\ \frac{1}{\sqrt{2}} \begin{bmatrix}1\\0\end{bmatrix}\end{bmatrix} = \begin{bmatrix}1\\&1\\&&1\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$





Feynman quantum circuit simulation



a0m2=1 0 \	q1m2= 0
q0m2= 0 >	q1m2= 1
q0m2= 1 >	q1m2= 0
	q1m2= 1

q0m1= 0 >		q0m1= 1 >	
q1m1= 0 >	q1m1= 1 >	q1m1= 0 >	q1m1= 1 >
1	0	0	0
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0	0	0	1
0	0	1	0

q0m0		q0m2
	q1m1	<u>q1m2</u>

q0m2= 0 >	q1m2= 0 >
	q1m2= 1 >
q0m2= 1 >	q1m2= 0 >
	q1m2= 1 >

$q0m0= 0\rangle$		$q0m0= 1\rangle$		
$q1m1= 0\rangle$	q1m1= 1 >	q1m1= 0 >	$q1m1= 1\rangle$	
$1/\sqrt{2}$	0	$1/\sqrt{2}$	0	
0	$1/\sqrt{2}$	0	$1/\sqrt{2}$	
0	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	
$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0	

Where we are going.

What are quantum variational algorithms?

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What is quantum circuit simulation?

Why are the conventional techniques insufficient?

How do we represent quantum circuits as logic formulas?

Why does it help with variational algorithm simulation, and by how much?

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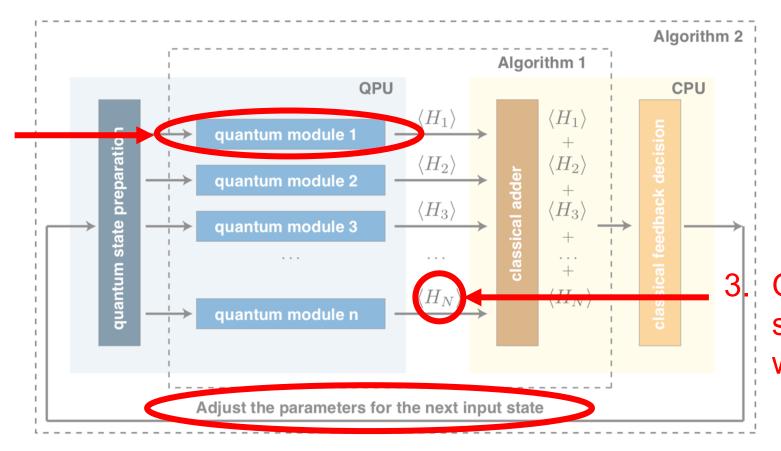
Why are the conventional techniques insufficient?

How do we represent quantum circuits as logic formulas?

• Why does it help with variational algorithm simulation, and by how much?

The unique challenge of simulating noisy variational algorithms

1. Needs to simulate noise, and quantum circuits are wide but shallow



Only need samples, not full wavefunctions.

2. Require repeated simulation with different parameters

Our toolchain: Bayesian network knowledge compilation for noisy quantum circuit simulation and sampling

1. Noisy quantum circuits to Bayesian network

 Needs to simulate noise

- 2. Bayesian networks to conjunctive normal form (CNF)
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Repeated simulation with different parameters

- 4. Exact inference on AC for quantum circuit simulation
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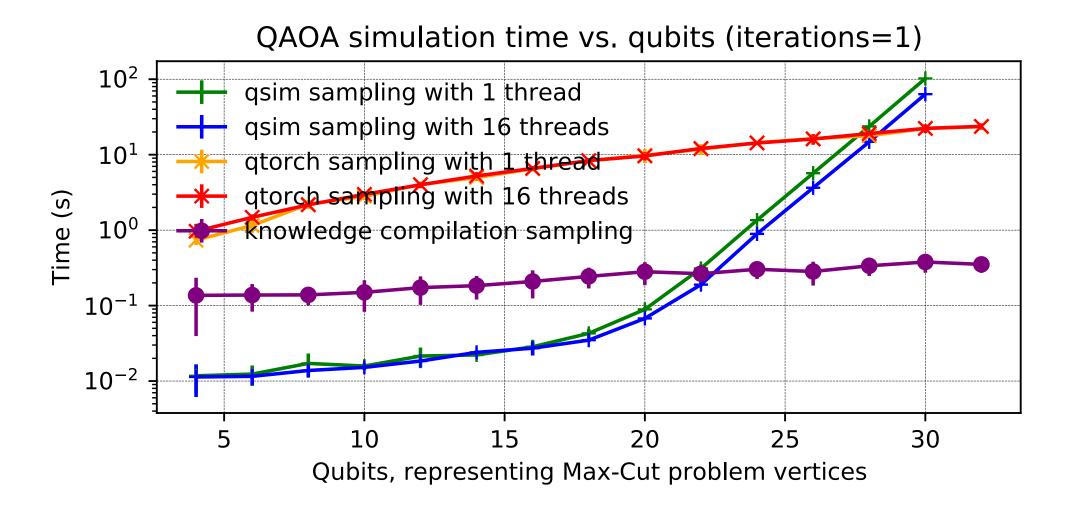
Result 1: It works!

With minimal modification, knowledge compilation exact inference can be repurposed for quantum simulation

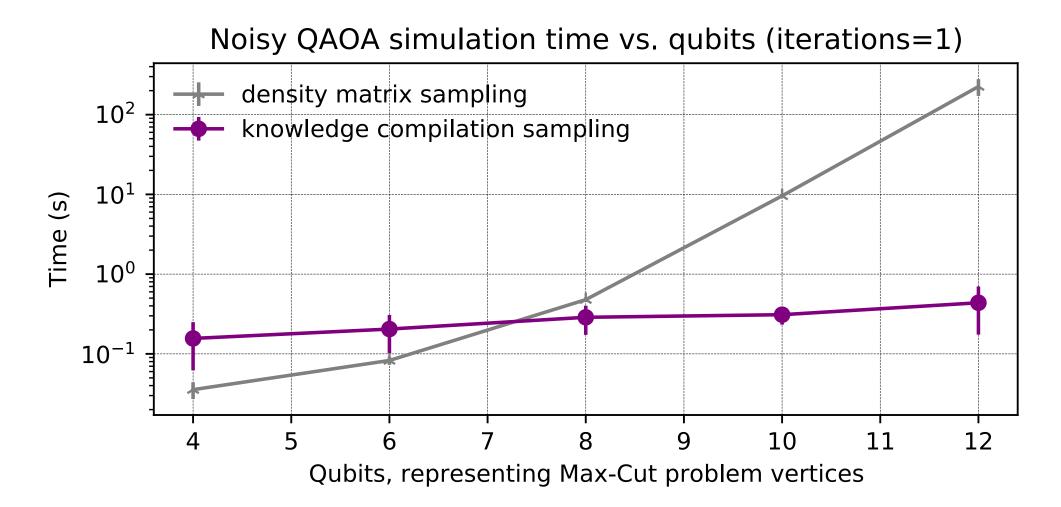
 Can accurately simulate Pauli gates, CNOT, CZ, phase kickback, Toffoli, CHSH protocol, Deutsch-Jozsa, Bernstein-Vazirani, hidden shift, quantum Fourier transform, Shor's, Grover's...

Passes Google Cirq's suite of test harness for quantum simulators

Result 2: Ideal circuit simulation



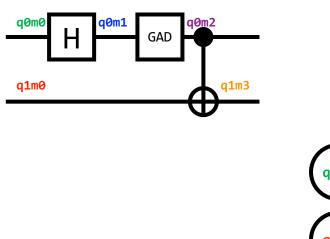
Result 2: Noisy circuit simulation



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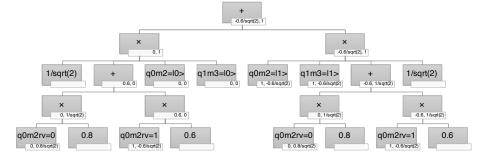
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q0m0 q0m1 q0m2 q1m3

The Hadamard gate:

```
q0m0=|0> AND q0m1=|0> -> +1/sqrt(2)
q0m0=|0> AND q0m1=|1> -> +1/sqrt(2)
q0m0=|1> AND q0m1=|0> -> +1/sqrt(2)
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```



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Broader research agenda: new representations for quantum computing

Schrödinger: state vectors and density matrices

Heisenberg: stabilizer formalism

Feynman: tensor-network path sums

Binary decision diagrams (new?)

Logical satisfiability equations (this work; new?)

Broader research agenda: new representations for quantum computing

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Binary decision diagrams (new?)

potential synergies

Logical satisfiability equations (this work; new?)