Quantum computer architecture

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Where we are in the semester

Full stack quantum computer engineering

1. Fundamentals: superposition and entanglement
2. Canonical algorithms: Shor’s factoring algorithm
3. NISQ Algorithms: QAOA & VQE
4. Google Cirq, IBM Qiskit
5. Programming languages, representations
6. Extracting success: quantum computer architecture
7. Prototypes: quantum computer microarchitecture
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Challenges of quantum computer architecture

Scheduling

Qubit mapping

Topological constraints resolving
The SWAP gate decomposes to three CNOT gates

Physical-gate decomposition
From quantum programs to quantum computer operation

Quick summary of the steps so far
- Loop unrolling
- Module flattening
- Logical-level optimization

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Physical-gate decomposition
Scheduling for maximum parallelism

Some types of gates commute, so we can move earlier or later.

**Figure:** Credit: [Alam et al., 2020]
Parallelism constraints

1. Amount of parallelism available in the instruction stream
2. Achievable parallelism in the control microarchitecture ("each student gets one coaxial input")
3. Safe parallelism despite crosstalk due to spatiotemporal and spectral overlap
From quantum programs to quantum computer operation

Next steps

- Qubit mapping
- Topological constraints resolving
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Physical-gate decomposition
Qubit mapping

- Ion trap qubits: fully connected topology
- Superconducting qubits: arbitrary qubits cannot directly interact; needs chain of swap gates

Fig. 1. Examples of several IBM cloud accessible devices. The top left 5-qubit device was the first one made available via the IBM Quantum Experience [40]. The one to the right of it was made available after including additional entangling gates between two pairs of qubits. A 16-qubit device was made available approximately a year after the first device. The devices in the bottom row show three variations of 20-qubit devices available to members of the IBM Q Network [41].

**Figure:** Credit: [Córcoles et al., 2020]
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Physical-gate decomposition
Topological constraints resolving

**Figure 3.** (a) Layout of a 6-qubit quantum computer, (b)-(e) are possible routes from A to F. Note that options (b)(c)(d) have an identical number of swaps and (e) incur higher swaps. An intelligent policy would choose one from (b)(c)(d).

**Figure:** Credit: [Tannu and Qureshi, 2019]

Superconducting qubits: arbitrary qubits cannot directly interact; needs chain of swap gates
Topological constraints resolving

Figure 3. (a) SWAP Gate Decomposition, (b) Physical Qubit Coupling Graph Example, (c) Original Quantum Circuit, (d) Updated Hardware-Compliant Quantum Circuit

Figure: Credit: [Li et al., 2019]

Superconducting qubits: arbitrary qubits cannot directly interact; needs chain of swap gates
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**Topological constraints resolving**

The SWAP gate decomposes to three CNOT gates

Physical-gate decomposition
The SWAP gate decomposes to three CNOT gates

The SWAP gate is

\[
\text{SWAP}_{0,1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

This unitary matrix realizes the following transformation:

- \(|00\rangle \rightarrow |00\rangle\)
- \(|01\rangle \rightarrow |10\rangle\)
- \(|10\rangle \rightarrow |01\rangle\)
- \(|11\rangle \rightarrow |11\rangle\)
The SWAP gate decomposes to three CNOT gates

The CNOT\(_{0,1}\) gate

The CNOT gate with the zeroth qubit as control, first qubit as target is:

\[
\text{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]

This unitary matrix realizes the following transformation:

- \(\ket{00} \rightarrow \ket{00}\)
- \(\ket{01} \rightarrow \ket{11}\)
- \(\ket{10} \rightarrow \ket{10}\)
- \(\ket{11} \rightarrow \ket{10}\)

The CNOT\(_{1,0}\) gate

The CNOT gate with the first qubit as control, zeroth qubit as target is:

\[
\text{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\]

This unitary matrix realizes the following transformation:

- \(\ket{00} \rightarrow \ket{00}\)
- \(\ket{01} \rightarrow \ket{01}\)
- \(\ket{10} \rightarrow \ket{11}\)
- \(\ket{11} \rightarrow \ket{01}\)
The SWAP gate decomposes to three CNOT gates

\[
\text{CNOT}_{0,1} \text{CNOT}_{1,0} \text{CNOT}_{0,1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} = \text{SWAP}_{0,1}
\]
Topological constraints resolving

Superconducting qubits: arbitrary qubits cannot directly interact; needs chain of swap gates

Figure 3. (a) SWAP Gate Decomposition, (b) Physical Qubit Coupling Graph Example, (c) Original Quantum Circuit, (d) Updated Hardware-Compliant Quantum Circuit

Figure: Credit: [Li et al., 2019]
From quantum programs to quantum computer operation

Next steps
- Physical-gate decomposition
- Physical-level optimization
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Physical-gate decomposition
Physical-gate decomposition

\[ |0\rangle \xrightarrow{R(\theta,\phi)} \cos \left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi \sin \left(\frac{\theta}{2}\right)} |1\rangle \]

\[ |1\rangle \xrightarrow{R(\theta,\phi)} \cos \left(\frac{\theta}{2}\right) |1\rangle - e^{-i\phi \sin \left(\frac{\theta}{2}\right)} |0\rangle \]

\[ |x\rangle \xrightarrow{R(\theta,\phi)} |\tilde{x}\rangle \]

\[ |x_C\rangle \xrightarrow{\text{CNOT}} |x_C\rangle \]

\[ |x_T\rangle \xrightarrow{\text{XOR}} |\tilde{x}_T\rangle \]

(a)

(b)

FIG. 1. The rotation and controlled-NOT (CNOT) gates are an example of a universal quantum gate family when available on all qubits, with explicit evolution (above) and quantum circuit block schematics (below). (a) The single-qubit rotation gate \( R(\theta, \phi) \), with two continuous parameters \( \theta \) and \( \phi \), evolves input qubit state \( |x\rangle \) to output state \( |\tilde{x}\rangle \). (b) The CNOT (or reversible XOR) gate on two qubits evolves two (control and target) input qubit states \( |x_C\rangle \) and \( |x_T\rangle \) to output states \( |\tilde{x}_C = x_C\rangle \) and \( |\tilde{x}_T = x_C \oplus x_T\rangle \), where \( \oplus \) is addition modulo 2, or equivalently the XOR operation.

Figure: Credit: [Alexeev et al., 2020]

- Clifford + T ISA is sensible for an error-corrected machine
- But for NISQ machine, best two-qubit gate is dependent on native gate set
BOX 2. CHOOSING A CNOT GATE DECOMPOSITION

A logical controlled-NOT, or CNOT, gate is an entangling operation that flips the target qubit between 1 and 0 if the control qubit is in the 1 state. It must be decomposed into a sequence of native quantum gates for the qubit technology to perform the gate operation on the specific qubit system. Two possible decompositions are shown, where RX, RY, and RZ denote rotations around the x-, y- and z-axes, respectively. A system performance simulation could provide metrics to help choose which CNOT to incorporate into the specific design.

 Depending on the fidelity of the single- and two-qubit gates in the circuits and on their speed, researchers may want to choose only one to implement the logical CNOT in the system. Depending on the performance of the qubits available at a particular point in the execution of the algorithm, it is also possible to choose a different logical CNOT sequence.

Figure: Credit: [Matsuura et al., 2019]
Physical-gate decomposition

**Figure 1.** Hardware qubit technology, native gate set, and software-visible gate set in the systems used in our study. Each qubit technology lends itself to a set of native gates. For programming, vendors expose these gates in a software-visible interface or construct composite gates with multiple native gates.

**Figure:** Credit: [Murali et al., 2019]

Two qubit gates remain dominant sources of errors.
Primary sources

- [Ding and Chong, 2020, Chapters 4, 6, 7]
- [Córcoles et al., 2020, Section III.B]
- [National Academies of Sciences, Engineering, and Medicine, 2019, Chapter 6.5]
- [Martonosi and Roetteler, 2019, Chapter 6]


A systems perspective of quantum computing.
*PhT*, 72(3):40–46.


