Quantum computing fundamentals: single qubits, multiple qubits

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September 12, 2022

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Functional completeness

All computation on binary variables can be represented as

$$f(x) = y$$

$$x \in \{0,1\}^n; y \in \{0,1\}^m$$

All Boolean expressions can be phrased as either CNF (and of ors) or DNF (or of ands).

Various sets of logic gates are functionally complete

- {NOT,AND,OR}
- {NAND}
- {NOR}

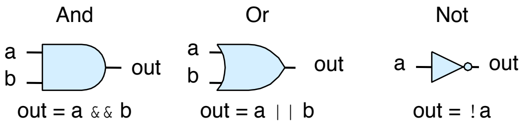

Figure: Source: CS:APP

The physics of information

What are the limits of computation energy efficiency?

Reversible operations

Toffoli (CCNOT) gate can represent all classical computation

CCNOT implements NAND

- Write down truth table for NAND.
- Write down truth table for CCNOT.
- Feed $|1\rangle$ into target qubit.

Creating classical computers out of purely reversible logic is a way to push the extremes of computing energy efficiency.

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Postulates of quantum mechanics

1. State space
2. Composite systems
3. Evolution
4. Quantum measurement

Quantum postulate 1: State space

The position or momentum of a physical system is described as a wavefunction

- Assuming continuous state space:

$$|\psi\rangle = \int_{-\infty}^{\infty} \psi(x)\, |x\rangle\, dx$$

$$\psi(x) \in \mathbb{C}$$

- Assuming discrete state space:

$$|\psi\rangle = \sum_{i=0}^{\infty} \psi(x_i)\, |x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

Quantum postulate 1: State space

The position or momentum of a physical system is described as a wavefunction

- Assuming discrete state space:

$$|\psi\rangle = \sum_{i=0}^{\infty} \psi(x_i)\,|x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

- Assuming discrete binarized state space:

$$|\psi\rangle = \sum_{i=0}^{1} \psi(x_i)\,|x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

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The state of a single qubit

Single qubit state

- $\alpha |0\rangle + \beta |1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$
- Amplitudes $\alpha, \beta \in \mathbb{C}$
- $|\alpha|^2 + |\beta|^2 = 1$

Many physical phenomena can be in superposition and encode qubits

- Polarization of light in different directions
- Electron spins (Intel solid state qubits)
- Atom energy states (UMD, IonQ ion trap qubits)
- Quantized voltage and current (IBM, Google superconducting qubits)

If multiple discrete values are possible (e.g., atom energy states, voltage and current), we pick (bottom) two for the binary abstraction.

Bloch sphere

Representation of pure states of a single qubit

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

- θ polar angle
- ϕ azimuthal angle

Euler's formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

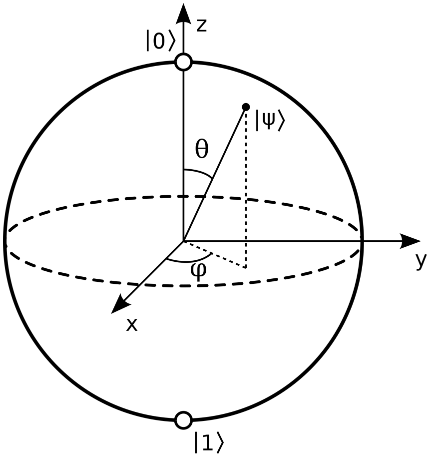


Figure: Source: Wikimedia

Bloch sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Important locations on the Bloch sphere

- $|+\rangle = H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
- $|-\rangle = H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

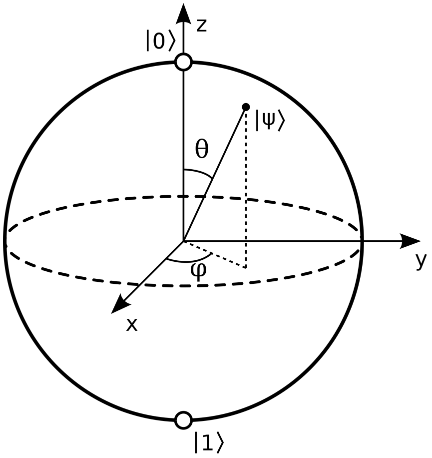


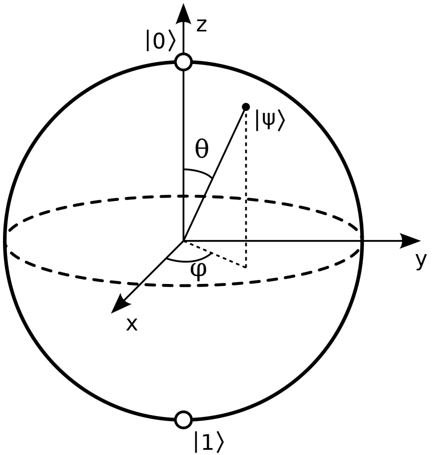
Figure: Source: Wikimedia

Bloch sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Rotations around the Bloch sphere



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