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Bloch Sphere Representation Of Noisy States

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What is Noise?

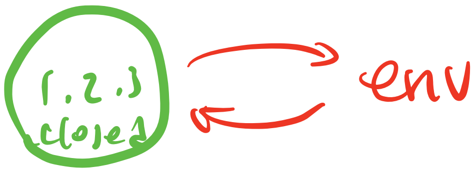
- Quantum noise is anything that can cause a quantum computer to malfunction.
- In the domain of quantum computing, **noise significantly affects the accuracy of measurements**, which reduces the system's reliability, efficiency, and usefulness.
- When a qubit in a quantum computer gets affected by such kind of noise, the **information in it gets degraded**, just how sound quality is degraded by interference in a phone call.This is called decoherence.

Sources of Quantum Noise

1. **Heisenberg uncertainty**: According to the uncertainty principle, it is impossible to precisely measure both the position and momentum of a particle at the same time. This inherent uncertainty leads to quantum noise in measurements of these quantities.

2. **Noise in quantum states**: The quantum states of particles are inherently probabilistic, which means that they can exhibit random behavior. This randomness can lead to noise in measurements of these states.

3. **Electromagnetic signals from wifi, magnetic field of the earth**, etc.



Decoherence

- In quantum mechanics, **decoherence is the loss of coherence or correlation between the different parts of a quantum system**. In other words, it is the process by which a quantum system **transitions from a state of quantum behavior to classical behavior.**

- This typically happens when a **quantum system interacts with its environment in a way that causes it to become entangled with the surrounding particles**, effectively causing the system to lose its original quantum state.

- Decoherence is an important concept in quantum computing, as it can lead to errors in the calculations performed by a quantum computer.

The Density Matrix

- Matrix representation of possible quantum states weighted by respective probabilities
- Sum of multiplied probabilities and outer products
- Allows us to represent mixed states, such as those produced by noise

$$\rho \equiv \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

Example: |0> with ½ probability, |1> with ½ probability

$$\rho_B = \frac{1}{2}|0_B\rangle\langle 0_B| + \frac{1}{2}|1_B\rangle\langle 1_B|$$

$$= \frac{1}{2}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\rho_{|+\rangle} = 1|+\rangle\langle+|$$

$$= 1\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

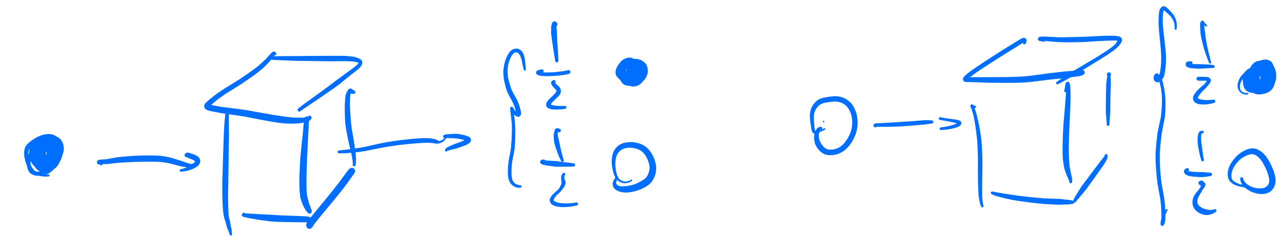
$$= \frac{I+X}{2}$$

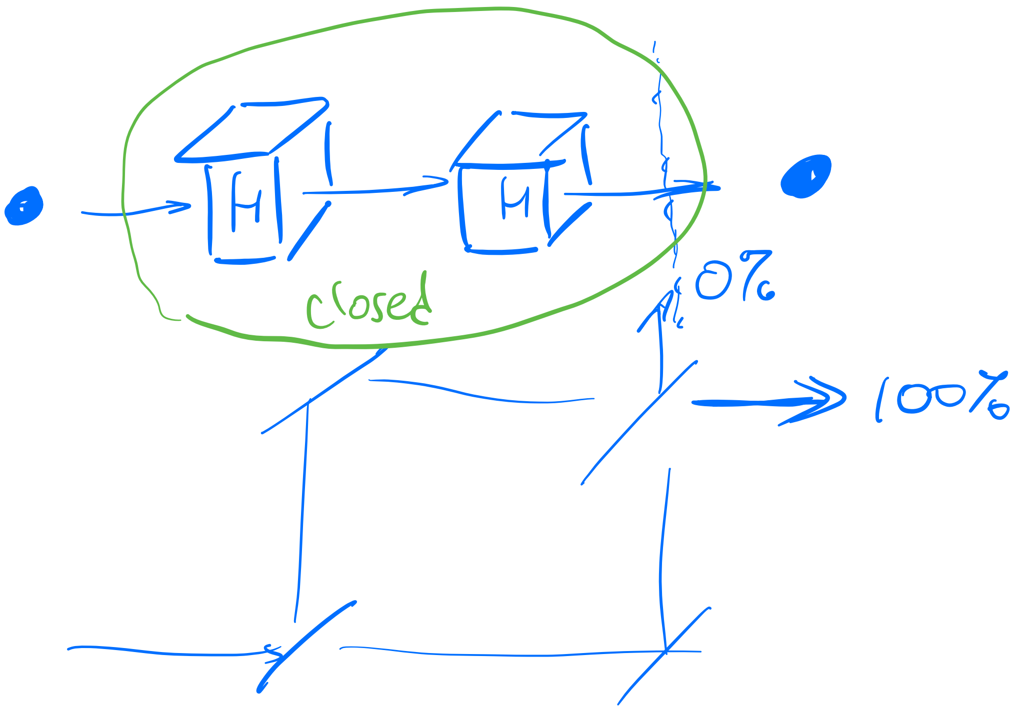
$$|+\rangle\langle+| \xrightarrow{\text{phase flip}\left(\frac{1}{2}\right)} \frac{1}{2}|+\rangle\langle+| + \frac{1}{2}|-\rangle\langle-|$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

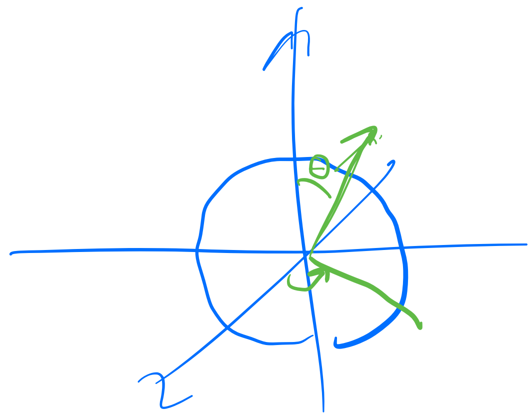
$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2} I$$

$$|0\rangle\langle 0| \xrightarrow{BF(p=.5)} \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$





$\alpha|0\rangle + \beta|1\rangle$ ⟶



$|\alpha|^2 + |\beta|^2 = 1$

Expansion of the Density Matrix

- In order to map a mixed state on the Bloch sphere using a density matrix, we use this expanded form of ρ, where r is the state vector rotated by σ. Solving yields our three coordinates on the sphere.

$$\rho = \frac{1}{2}\left(\hat{I} + \vec{r} \cdot \hat{\vec{\sigma}}\right)$$

$$\rho = \frac{1}{2}\hat{I} + \frac{1}{2}r_x\hat{\sigma}_x + \frac{1}{2}r_y\hat{\sigma}_y + \frac{1}{2}r_z\hat{\sigma}_z,$$

where $\hat{I}$ is the identity matrix, and $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ the X, Y, Z Pauli matrices, respectively:

$$\hat{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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Kraus Operators

- The Kraus operators are a **set of mathematical operators that are used to describe the effects of a quantum channel** on a quantum system.
- Each term K_α is a Kraus operator - not necessarily unitary but trace-preserving
  - trace-preserving: $\sum_\alpha K_\alpha K_\alpha^\dagger = I$
  - recall that this is similar to unitary operators - unitary channels are the case when there is only one Kraus operator
- Noise channels can be written as a sum of Kraus operators. These can be mathematically derived or produced with functions in Cirq, Qiskit etc.

$$\rho' = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

Summary of Representations

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

State Vector - simplest, good for pure states

$$\rho \equiv \sum_j p_j|\psi_j\rangle\langle\psi_j|$$

Density Matrix - good for mixed states

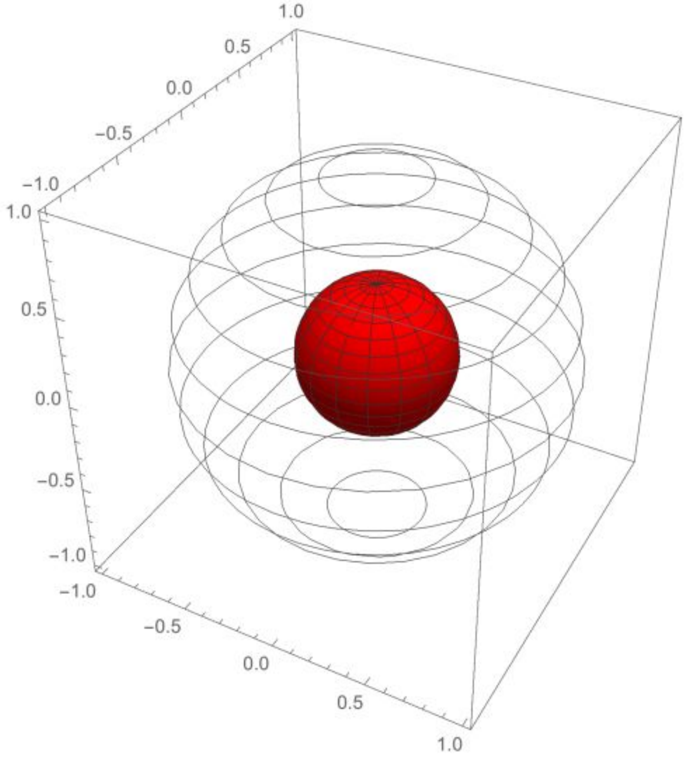
$$\rho = \frac{1}{2}\left(\hat{I} + \vec{r}\cdot\hat{\vec{\sigma}}\right)$$

Geometrical expansion of density matrix - good for plotting on sphere

$$\rho' = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$$

Kraus operators - describe quantum channels

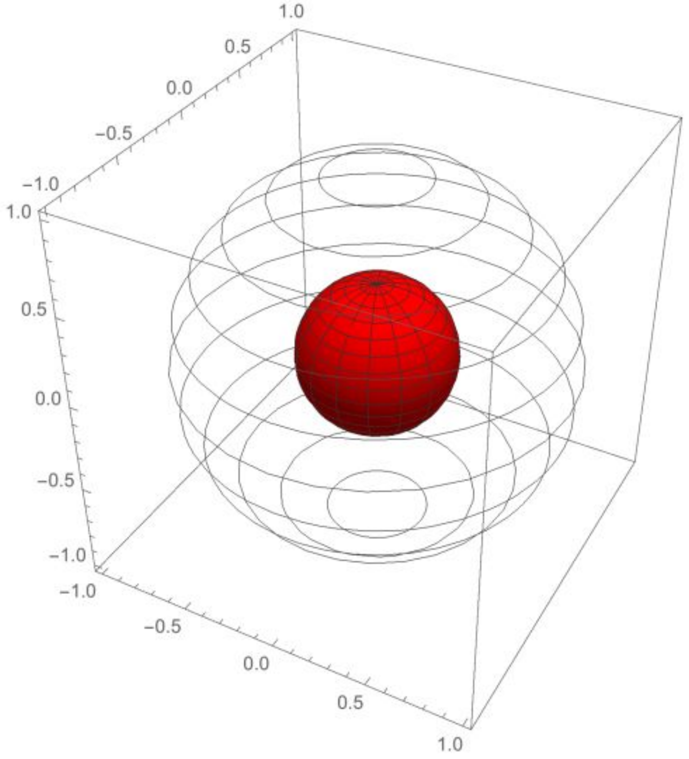
Depolarization



- Takes qubit to maximally mixed state I/2 with probability p -> all info about state is lost
  - maximally mixed state: density matrix is multiple of I
- No effect with probability 1-p

Effect on Bloch sphere: shrinking toward origin

Depolarization Equations



$$\rho' = p\frac{I}{2} + (1-p)\rho$$

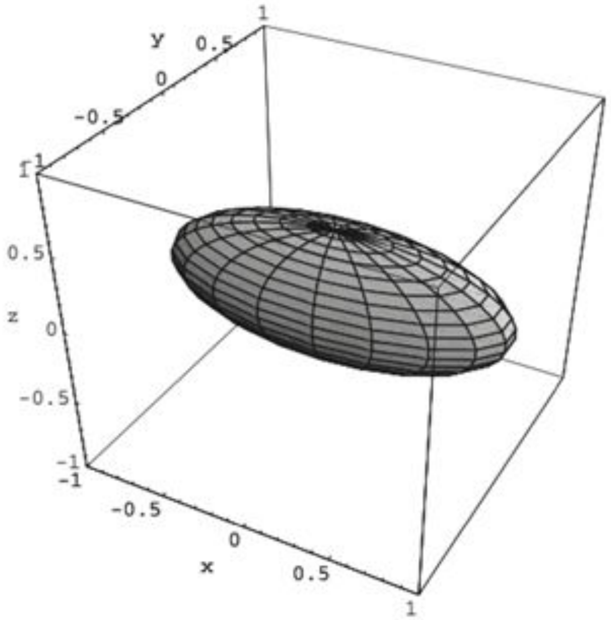
(same as the probability description from last slide)

We can rewrite this to get a Bloch vector:

$$\begin{aligned}\rho' &= p\frac{I}{2} + (1-p)\frac{1}{2}(I + \vec{r}\cdot\vec{\sigma}) \\ &= p\frac{I}{2} + \frac{(1-p)}{2}\vec{r}\cdot\vec{\sigma} \\ &= p\frac{I}{2} + \frac{1}{2}(I + \vec{r'}\cdot\vec{\sigma})\end{aligned}$$

So that $\vec{r'} = (1-p)\vec{r}$

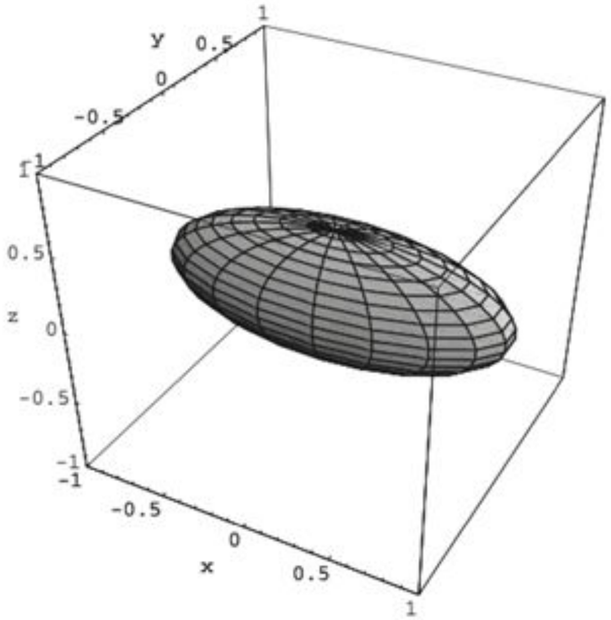
Bit Flip



- Flips bit with probability p
- No effect with probability 1-p

Effect on Bloch sphere: shrinks across y and z axes

Bit Flip Equations



$$\rho' = p\rho + (1-p)X\rho X$$

$$\rho' = \frac{1}{2}(I + p\vec{r}\cdot\vec{\sigma} + (1-p)X\vec{r}\cdot\vec{\sigma}X$$

Expanding on the last term gives us:

$$\begin{aligned} X\vec{r}\cdot\vec{\sigma}X &= X(r_x X + r_y Y + r_z Z)X \\ &= r_x X - r_y Y - r_z Z \end{aligned}$$

Finally giving us:

$$\begin{aligned} \rho' &= \frac{1}{2}(I + r_x X + (2p-1)r_y Y + (2p-1)r_z Z) \\ &= \frac{1}{2}(I + \vec{r'}\cdot\vec{\sigma}) \end{aligned}$$

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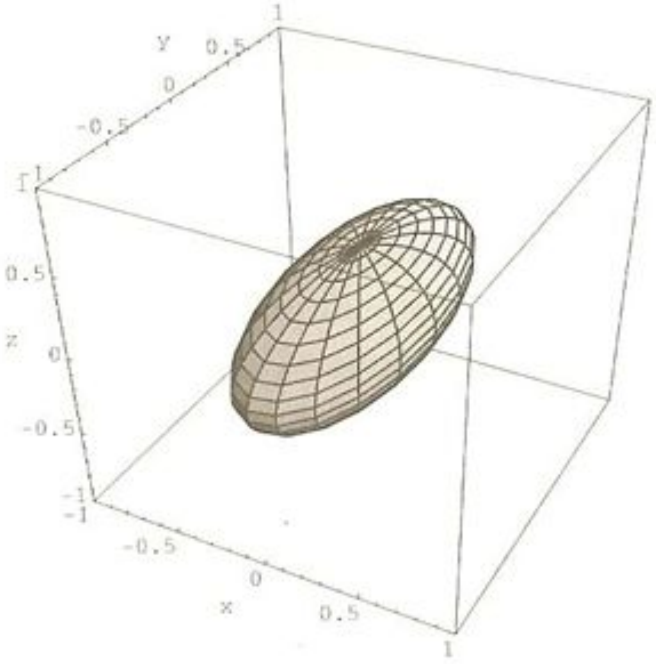
Bit Phase Flip



- Flips bit and sign with probability p
- No effect with probability 1-p

Effect on Bloch sphere: shrinks across x and z axes

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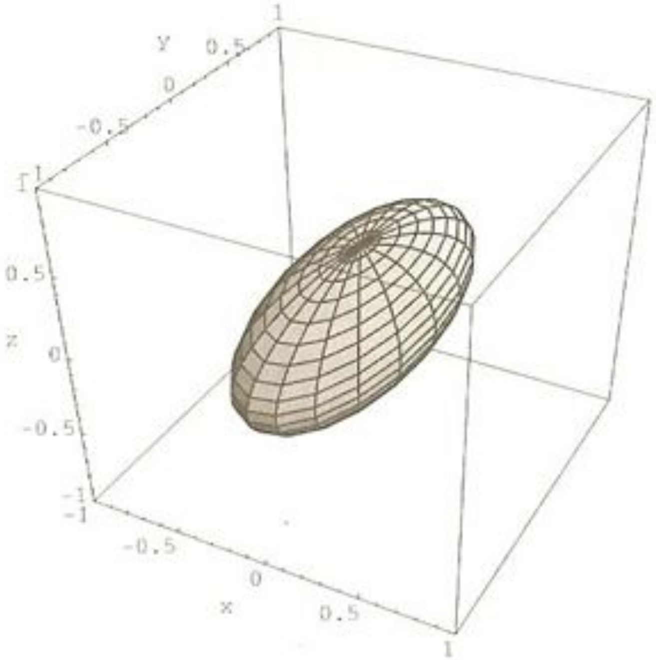
Bit Phase Flip Equations

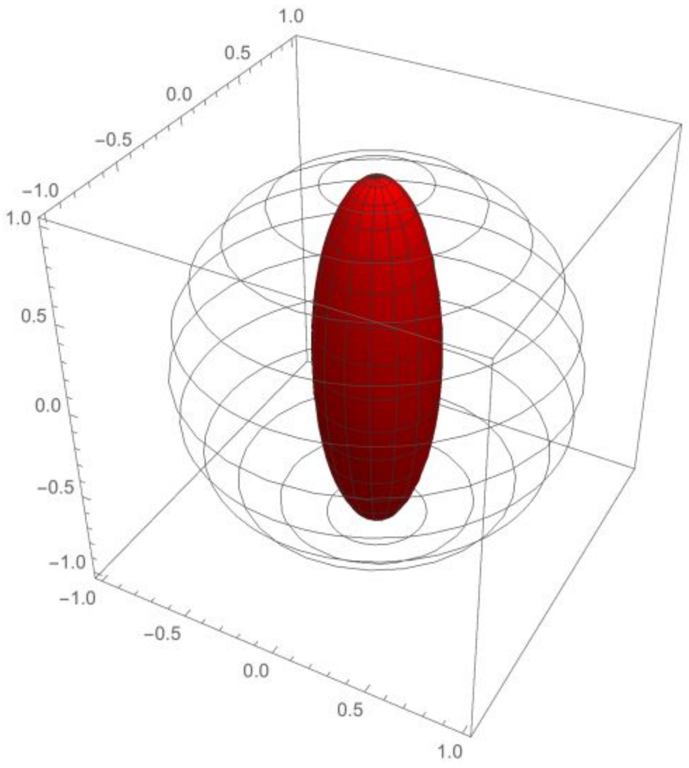


$$\rho' = p\rho + (1-p)Y\rho Y$$

As we can see, this is identical to bit flip but uses a Y gate. Intuitively we know this must affect only r_x and r_z, but it can be calculated the long way as well.

$$\rho' = \frac{1}{2}(I + (2p-1)r_x X + r_y Y + (2p-1)r_z Z)$$
$$= \frac{1}{2}(I + \vec{r'} \cdot \vec{\sigma})$$

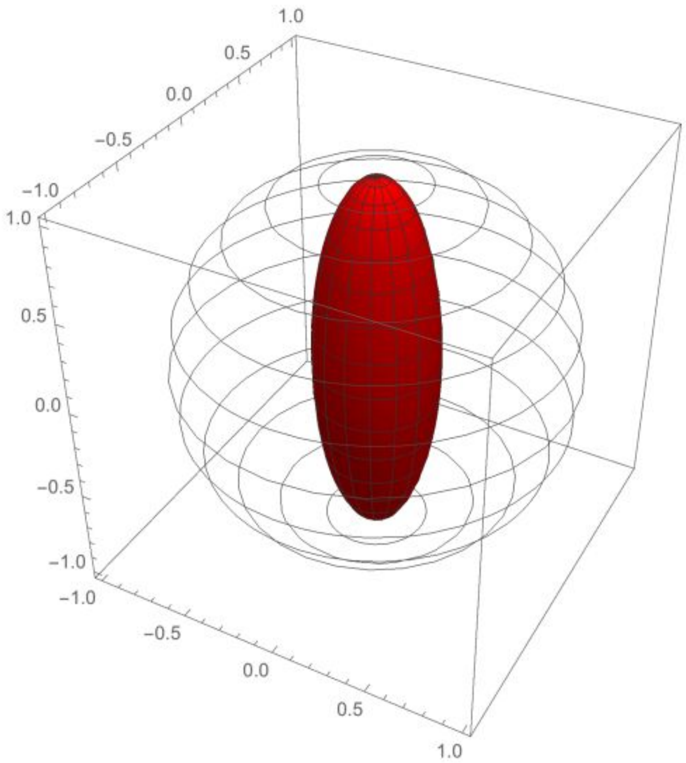
Phase Damping



- Flips sign with probability p
- No effect with probability 1-p

Effect on Bloch sphere: shrinks across x and y axes

Phase Damping Equations

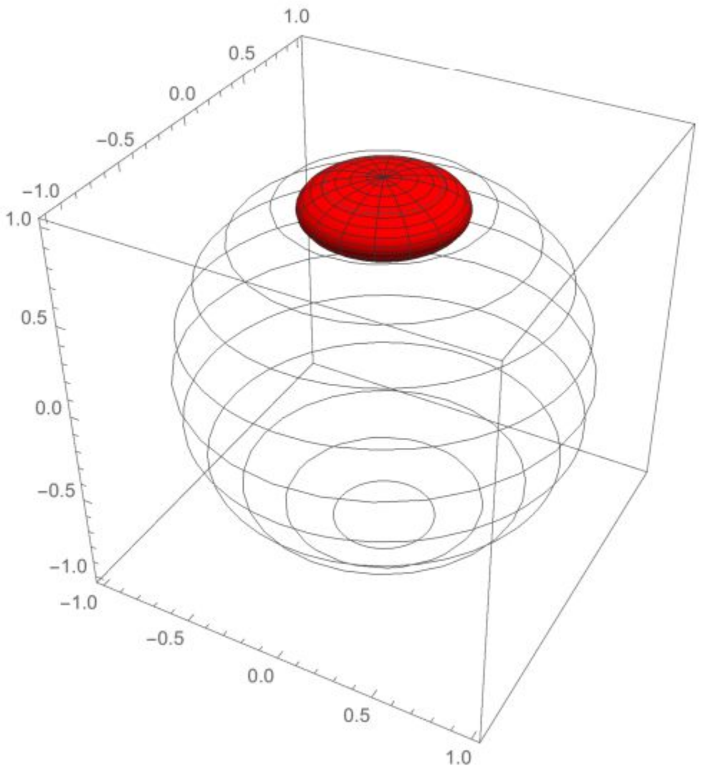


$$\rho' = p\rho + (1-p)Z\rho Z$$

As we can see, this is identical to bit flip but uses a Z gate. Intuitively we know this must affect only r_x and r_y, but it can be calculated the long way as well.

$$\rho' = \frac{1}{2}(I + (2p-1)r_x X + (2p-1)r_y Y + r_z Z)$$
$$= \frac{1}{2}(I + \vec{r'} \cdot \vec{\sigma})$$

Amplitude Damping



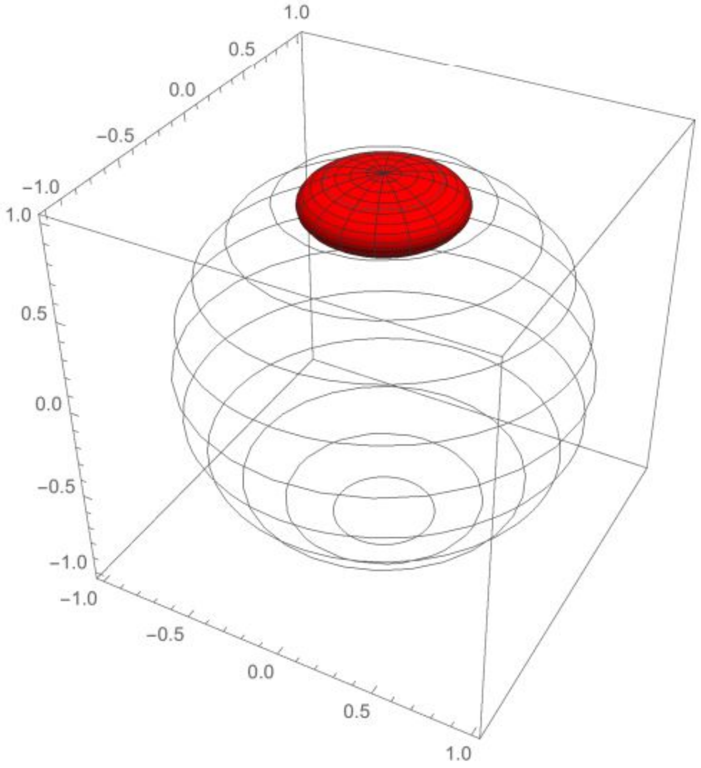
- Models loss of energy

  |0> -> |0> with probability 1

  |1> -> |0> with probability p
- Can be reversed to model gain of energy

Effect on Bloch sphere: shrinks height and width, center is shifted up or down

Amplitude Damping Equations



Amplitude damping is not unital: it shifts the center of the Bloch sphere, making it more complex

$$\rho' = K_0[\frac{1}{2}(I + \vec{r}\cdot\vec{\sigma})]K_0^\dagger + K_1[\frac{1}{2}(I + \vec{r}\cdot\vec{\sigma})]K_1^\dagger$$

(Expand on board)

$$\rho' = \frac{1}{2}(I + pZ + (\sqrt{1-p})r_x X + (\sqrt{1-p})r_y Y + (1-p)r_z Z)$$

Our Project Goals

- Simulating circuits with different noise channels
  - Visualize final state on Bloch sphere
  - Compare to noiseless circuit results to the ideal one

Questions

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References

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