The basics of logic design: Combinational logic

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Announcements

Class session plan

- Thursday, 4/18 & Tuesday, 4/23: Diving deeper: Digital logic. (CS:APP Chapter 4.2) (Recommended reading: Patterson & Hennessy, Computer organization and design, appendix on "The Basics of Logic Design." Available online via Rutgers Libraries)
- Thursday, 4/25: Survey of advanced topics in computer architecture.
- Tuesday, 5/7: 12:00-15:00, SERC 111, closed book, closed notes, no electronic devices, no calculator final exam.

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Computer organization

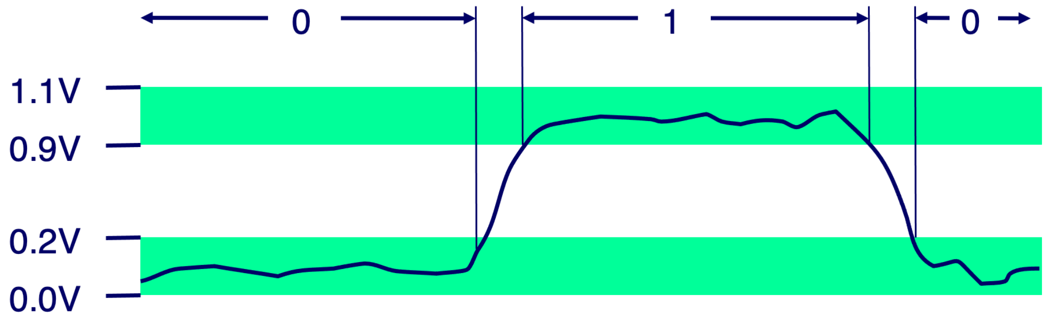
Layer cake

- Society
- Human beings
- Applications
- Algorithms
- High-level programming languages
- Interpreters
- Low-level programming languages
- Compilers
- Architectures
- Microarchitectures
- Sequential/combinational logic
- Transistors
- Semiconductors
- Materials science

Why binary

Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires

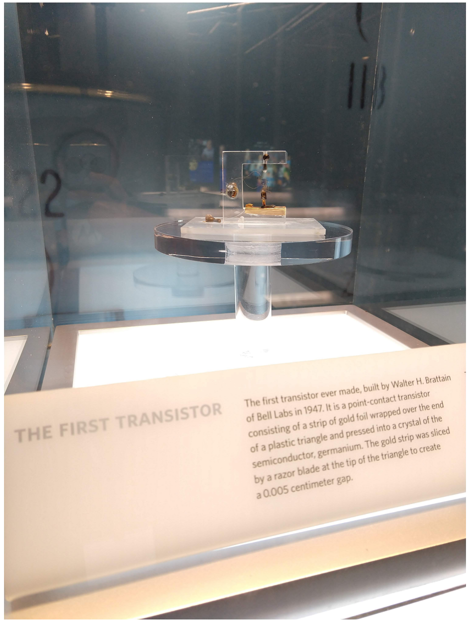


To build logic, we need switches

Vacuum tubes a.k.a. valves

Figure: Source: By Stefan Riepl (Quark48) - Self-photographed, CC BY-SA 2.0 https://commons.wikimedia.org/w/index.php?curid=14682022

Transistors



- The first transistor. Developed at Bell Labs, Murray Hill, New Jeresy
- https://www.bell-labs.com/about/locations/

MOSFETs

transistors { BJT, MOSFETs

MOS: Metal-oxide-semiconductor

- A sandwich of conductor-insulator-semiconductor.

FET: Field-effect transistor

- Gate exerts electric field that changes conductivity of semiconductor.

NMOS, PMOS, CMOS

PMOS: P-type MOS

- positive gate voltage, acts as open circuit (insulator)
- negative gate voltage, acts as short circuit (conductor)

NMOS: N-type MOS

- positive gate voltage, acts as short circuit (conductor)
- negative gate voltage, acts as open circuit (insulator)

CMOS: Complementary MOS

- A combination of NMOS and PMOS to build logical gates such as NOT, AND, OR.
- We'll go to slides posted in supplementary material to see how they work.

Combinational vs. sequential logic

Combinational logic

- No internal state nor memory
- Output depends entirely on input
- Examples: NOT, AND, NAND, OR, NOR, XOR, XNOR gates, decoders, multiplexers.

Sequential logic

- Has internal state (memory)
- Output depends on the inputs and also internal state
- Examples: latches, flip-flops, Mealy and Moore machines, registers, pipelines, SRAMs.

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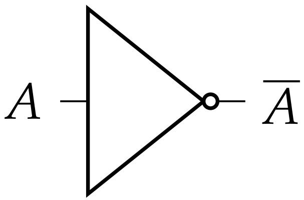
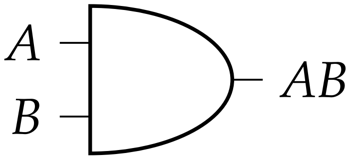
NOT gate

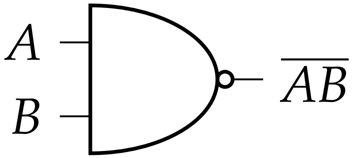


A	$\overline{A}$
0	1
1	0

Table: Truth table for NOT gate

AND gate, NAND gate





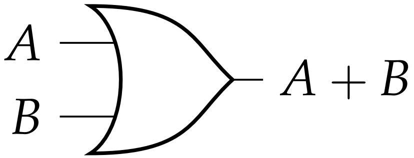
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

Table: Truth table for AND gate

A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

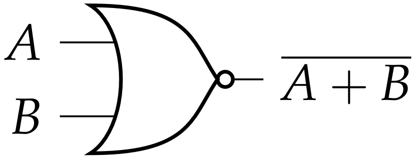
Table: Truth table for NAND gate

OR gate, NOR gate



A	B	$A+B$
0	0	0
0	1	1
1	0	1
1	1	1

Table: Truth table for OR gate



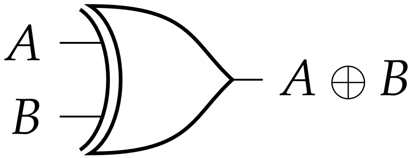
$= \neg(A+B)$

$= !(A||B)$

A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

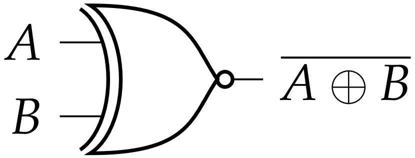
Table: Truth table for NOR gate

XOR gate, XNOR gate



A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

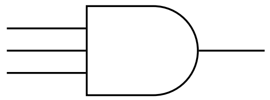
Table: Truth table for XOR gate

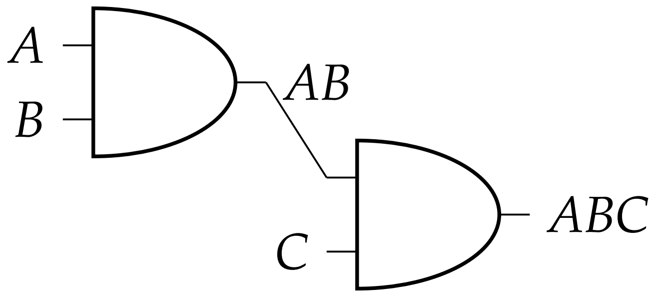


A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Table: Truth table for XNOR gate

More-than-2-input AND gate

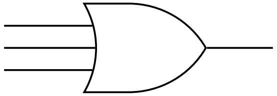


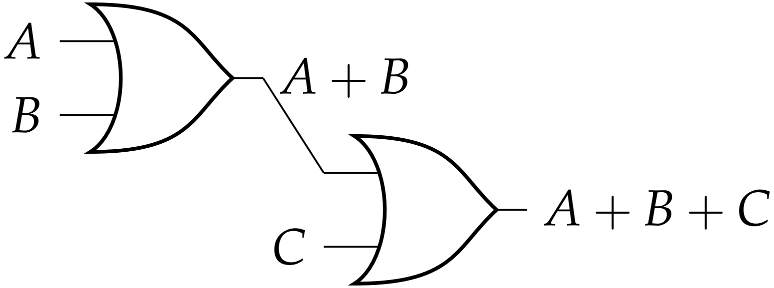


A	B	C	ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Table: Truth table for three-input AND gate

More-than-2-input OR gate





A	B	C	$A+B+C$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Table: Truth table for three-input OR gate

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The set of logic gates {NOT, AND, OR} is universal

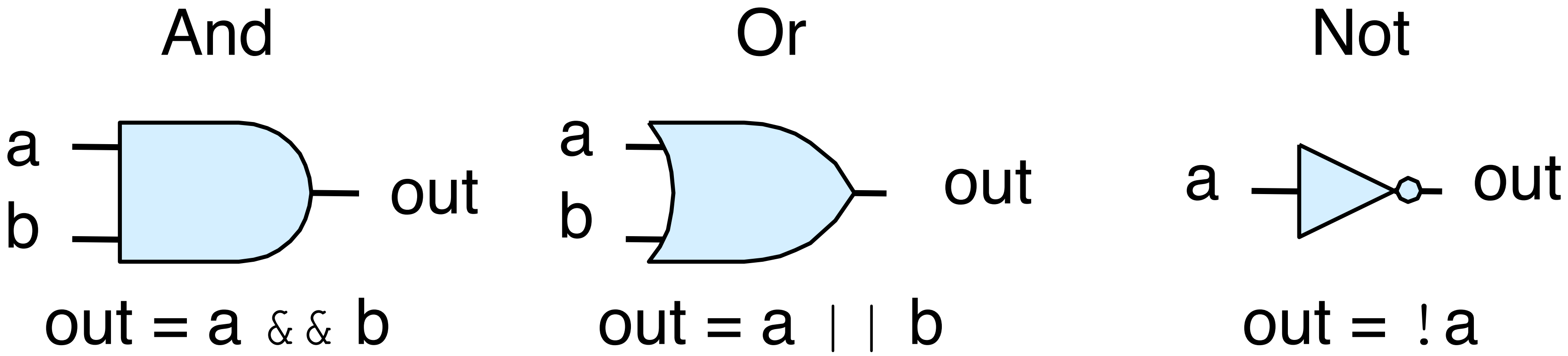


Figure: Source: CS:APP

The set of logic gates {NOT, AND, OR} is universal

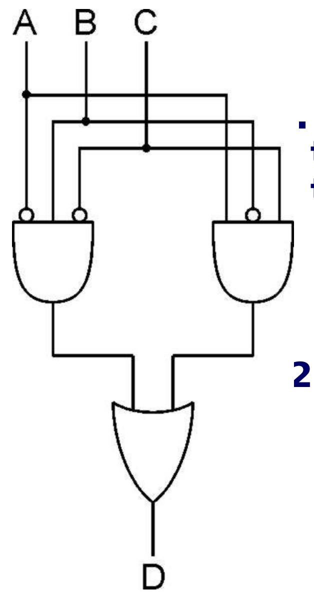
- Any truth table can be expressed as sum of products form.
- Write each row with output 1 as a product (minterm).
- Sum the products (minterm).
- Forms a disjunctive normal form (DNF).
- $D = \overline{A}B\overline{C} + A\overline{B}C$
- Always only needs NOT, AND, OR gates.
- Supplementary slides example...

B xor (A and c)

Logical Completeness

Can implement ANY truth table with AND, OR, NOT.

A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0



. AND combinations that yield a "1" in the truth table.

2. OR the results of the AND gates.

Sum of products
OR of AND clauses

The set of logic gates {NOT, AND, OR} is universal

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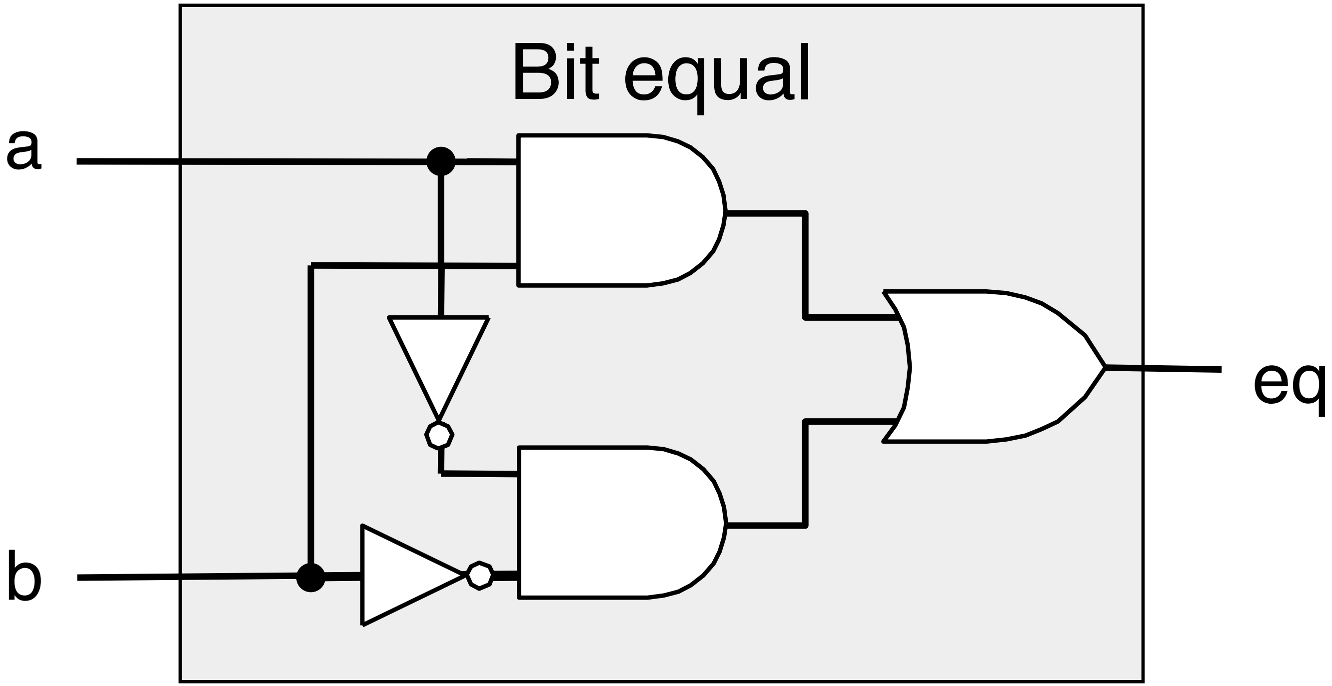
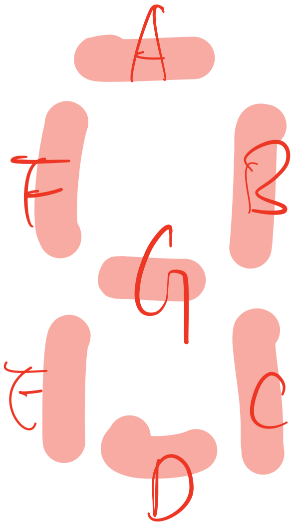


Figure: Source: CS:APP

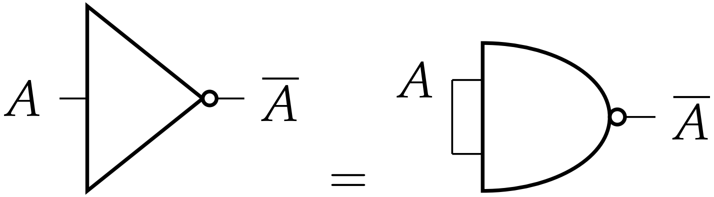


num	n[3]	n[2]	n[1]	n[0]	C
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1

$$\overline{C} = \overline{n[3]} \vee \overline{n[2]} \vee n[1] \vee \overline{n[0]}$$

The NAND gate is universal

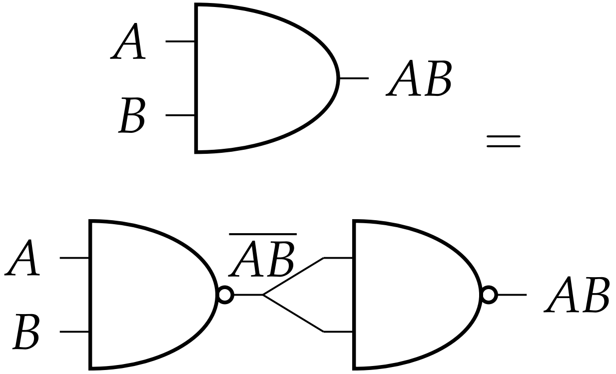
NOT gate as a single NAND gate



A	$\overline{A}$	AA	$\overline{AA}$
0	1	0	1
1	0	1	0

Table: $\overline{A} = \overline{AA}$

AND gate as two NAND gates



A	B	AB	$\overline{AB}$	$\overline{\overline{AB}}$
0	0	0	1	0
0	1	0	1	0
1	0	0	1	0
1	1	1	0	1

Table: $AB = \overline{\overline{AB}}$

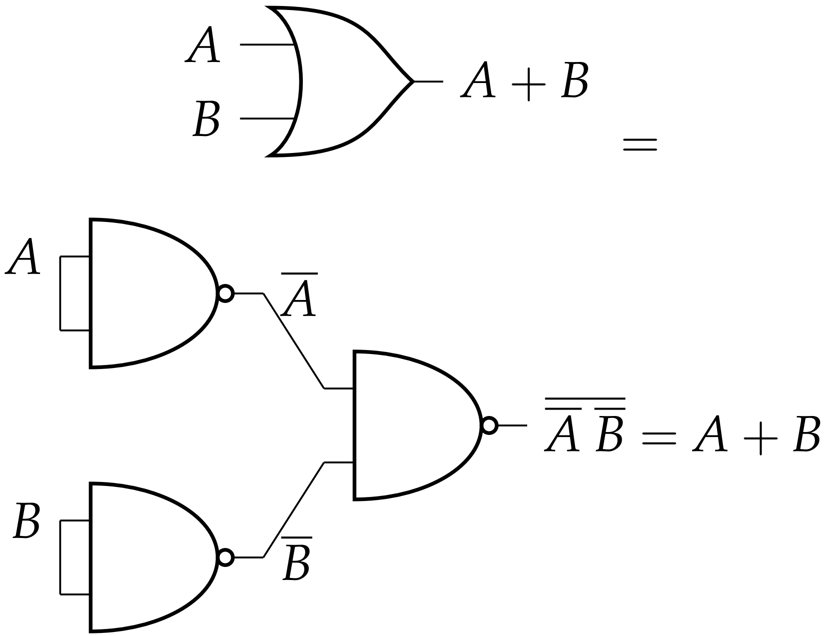
The NAND gate is universal

De Morgan's Law

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\,\overline{B}$	$A+B$	$\overline{A+B}$
0	0	1	1	1	0	1
0	1	1	0	0	1	0
1	0	0	1	0	1	0
1	1	0	0	0	1	0

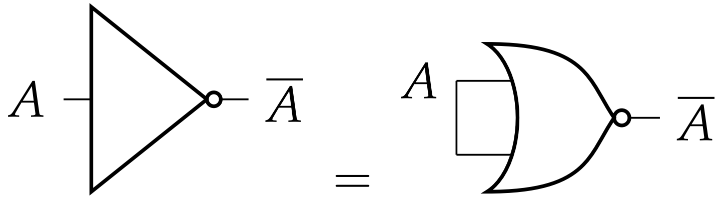
Table: $\overline{A}\,\overline{B} = \overline{A+B}$

OR gate as three NAND gates



The NOR gate is universal

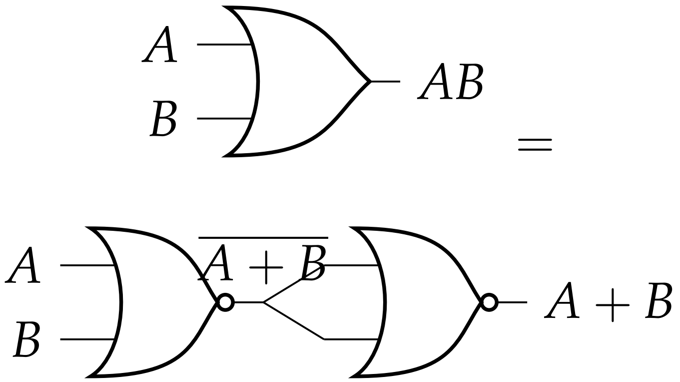
NOT gate as a single NOR gate



A	$\overline{A}$	$A+A$	$\overline{A+A}$
0	1	0	1
1	0	1	0

Table: $\overline{A} = \overline{A+A}$

OR gate as two NOR gates



A	B	$A+B$	$\overline{A+B}$	$\overline{\overline{A+B}}$
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	0	1

Table: $A+B = \overline{\overline{A+B}}$

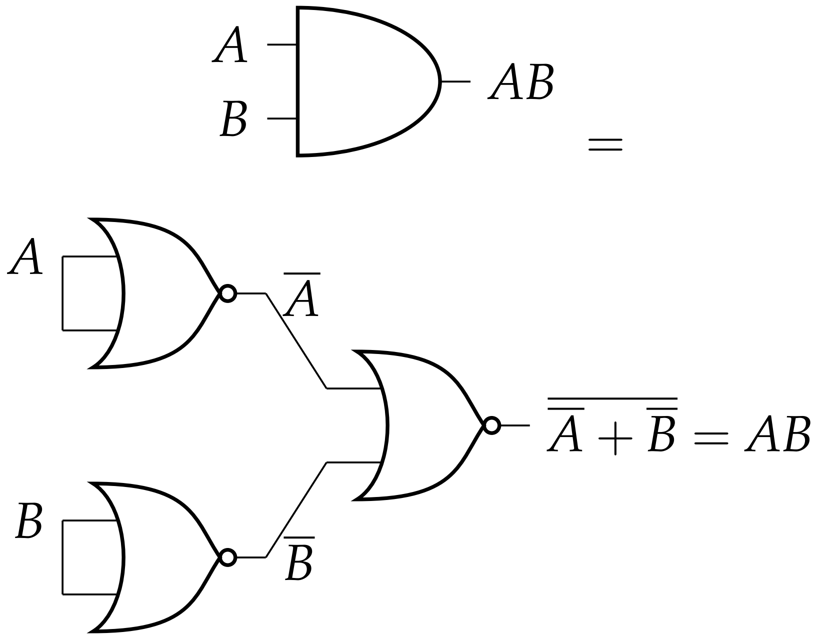
The NOR gate is universal

De Morgan's Law

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$	AB	$\overline{AB}$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

Table: $\overline{A}+\overline{B}=\overline{AB}$

AND gate as three NOR gates



Combinational vs. sequential logic

Combinational logic

- No internal state nor memory
- Output depends entirely on input
- Examples: NOT, AND, NAND, OR, NOR, XOR, XNOR gates, decoders, multiplexers.

Sequential logic

- Has internal state (memory)
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Decoders

Takes n-bit input, uses it as an index to enable exactly one of 2^n outputs

Internal design of 1:2 decoder

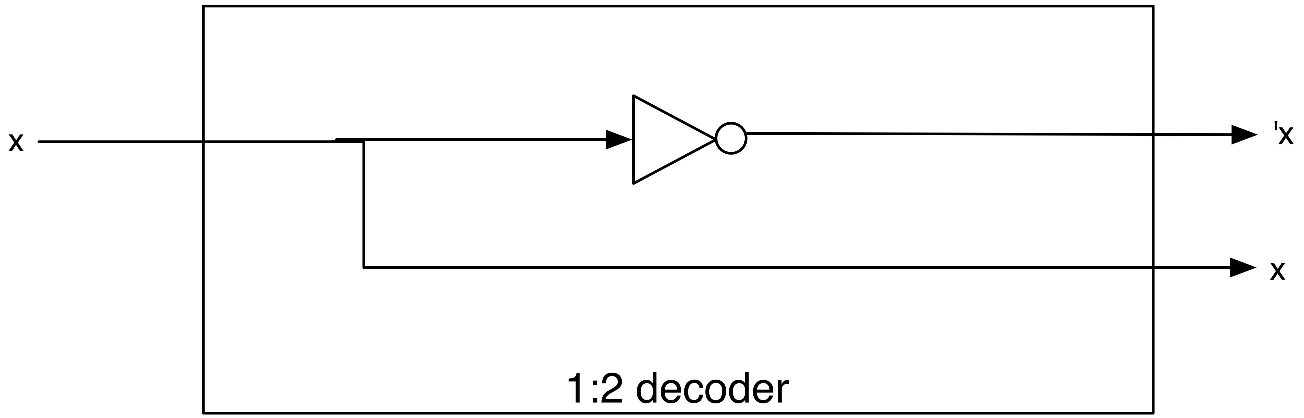


Figure: Source: Mano & Kime

Decoders

Hierarchical design of decoder (2:4 decoder)

Takes n-bit input, uses it as an index to enable exactly one of 2^n outputs

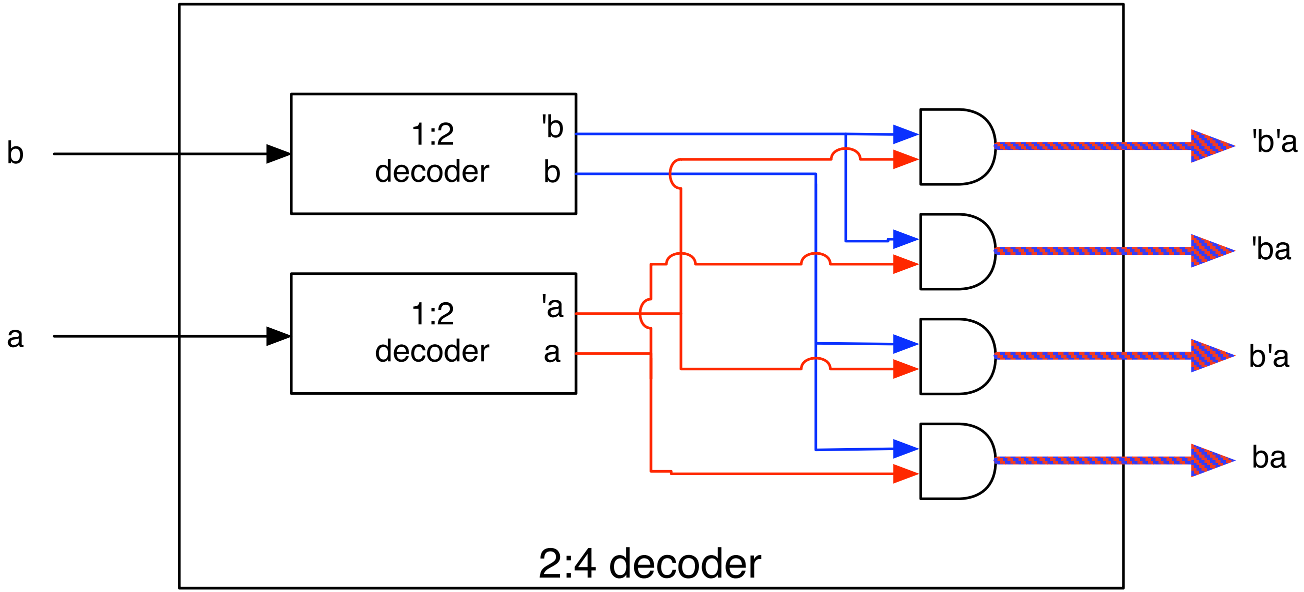


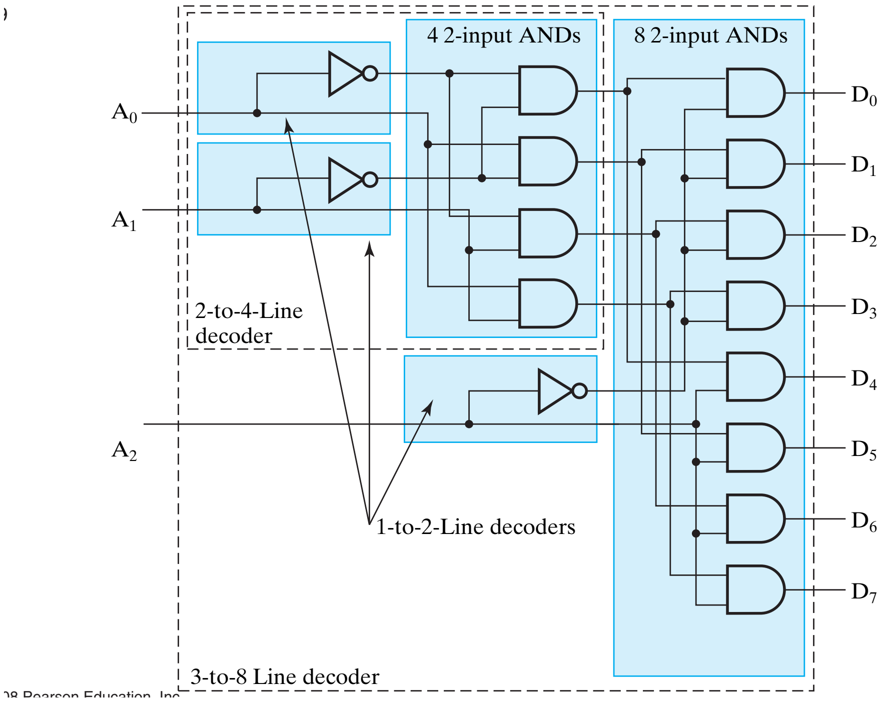
Figure: Source: Mano & Kime

Decoders

Decoder (3:8)

Hierarchical design: use small decoders to build bigger decoder

Takes n-bit input, uses it as an index to enable exactly one of 2^n outputs



Note: A_2 “selects” whether the 2-to-4 lin decoder is active in th top half (A_2=0) or the bottom (A_2=1)

Figure: Source: Mano & Kime

Multiplexers

Using n-bit selector input, select among one of 2^n choices

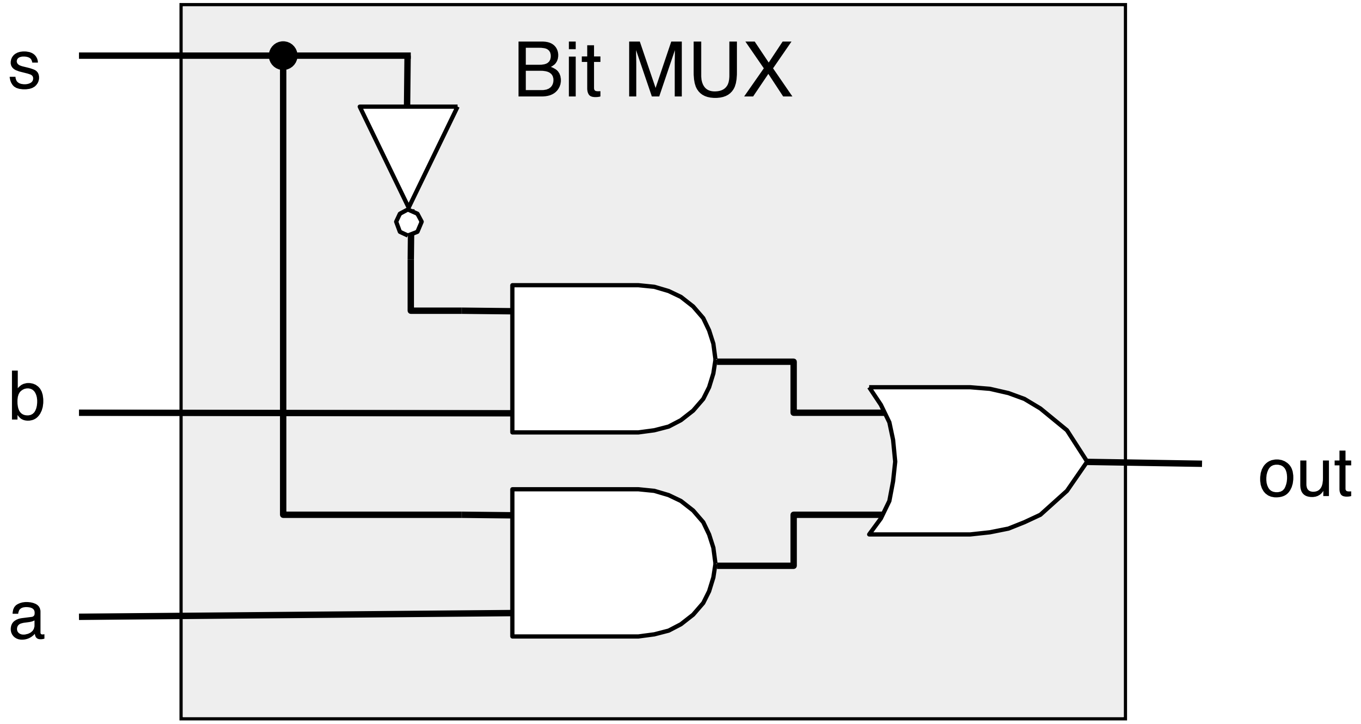


Figure: Source: CS:APP

Multiplexers

Using n-bit selector input, select among one of 2^n choices

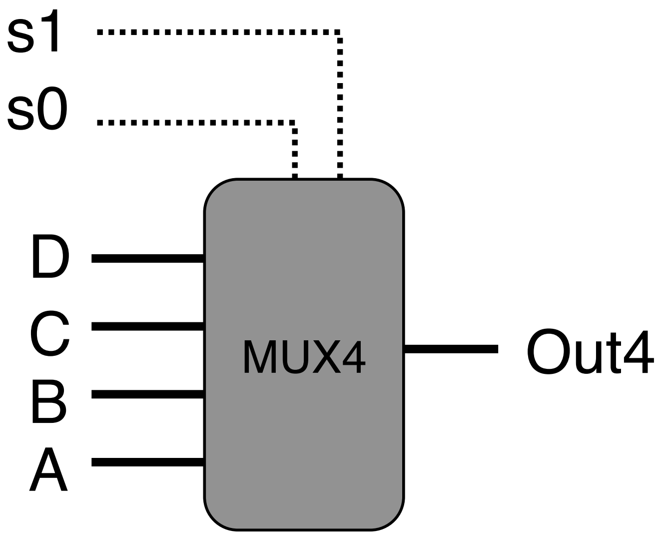


Figure: Source: CS:APP

Multiplexers

Using n-bit selector input, select among one of 2^n choices

Internal mux organization

3-26

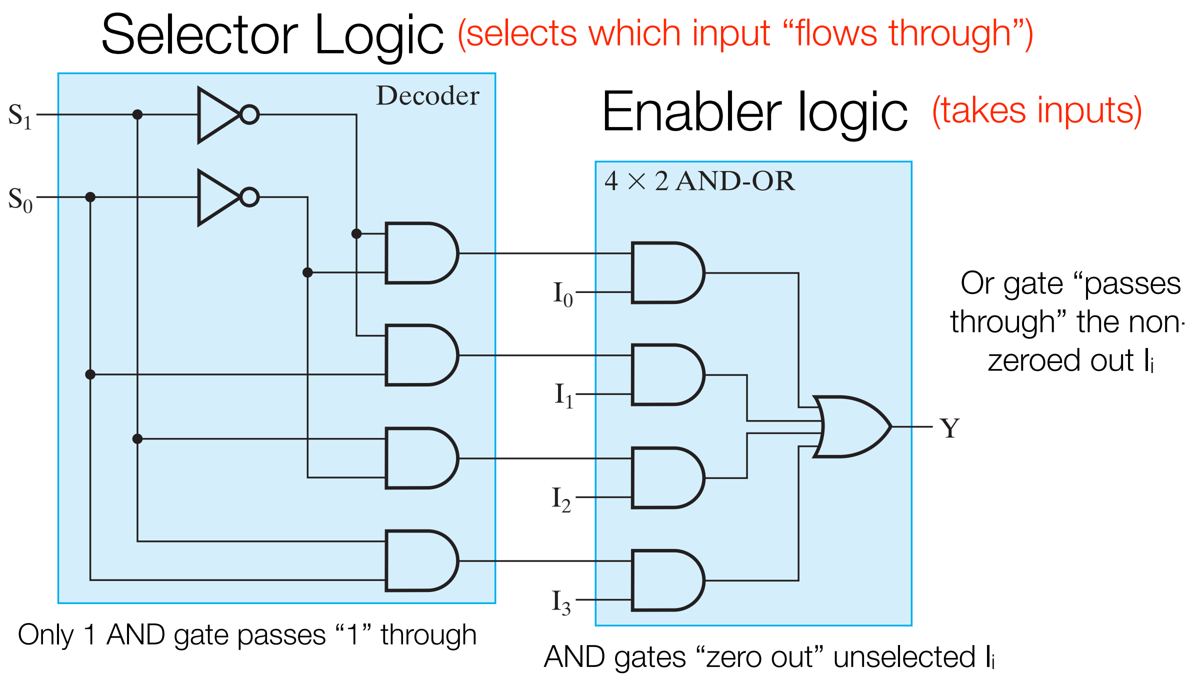


Figure: Source: Mano & Kime

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directMapped read logic