A Qudit Stabilizer Circuit Simulator

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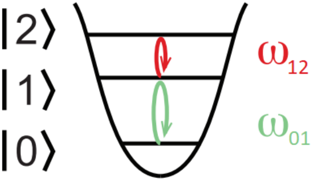
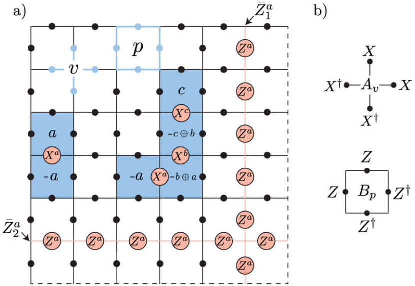
Motivation

- "Free" hardware currently unexploited by architects



- Quantum Error Correction



Qudits

- **Basis states** are **vectors**:

$$|0\rangle = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, |d-1\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

- **States**:

$$|\psi\rangle = \sum_{j=0}^{d-1} \alpha_j |j\rangle$$

where $\sum_{j=0}^{d-1} |\alpha_j|^2 = 1$.

- Qubits: $d = 2$. We are interested in $d > 2$.

Gates and Stabilizer States

- **Gates** evolve states: $|\psi\rangle \mapsto U|\psi\rangle$
- U made from $\{\mathbf{H}, \mathbf{P}, \mathbf{CNOT}, T\}$ when $d = 2$.
- Bolded gates are called **Clifford**.

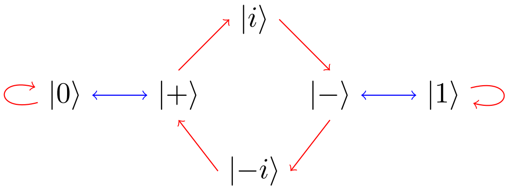


Figure: Single qubit stabilizer states. Blue arrows: H, Red arrows: P.

Pauli Gates and Tableaus: States as Tables

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{(Flip error)}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{(Phase error)}$$

H	$X \to Z$ $Z \to X$	
P	$X \to XZ$ $Z \to Z$	
CNOT	$X \otimes I \to X \otimes X$ $I \otimes X \to I \otimes X$ $Z \otimes I \to Z \otimes I$ $I \otimes Z \to Z \otimes Z$	

$$\left[\begin{array}{cccc|cccc} 1 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & & \ddots & 0 & 0 & & \ddots & 0 \\ 0 & \dots & \dots & 1 & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & 1 & \dots & \dots & 0 \\ 0 & & \ddots & 0 & 0 & & \ddots & 0 \\ 0 & \dots & \dots & 0 & 0 & \dots & \dots & 1 \end{array}\right]$$

The Key Idea

- Clifford and Pauli gates for $d > 2$:

$$H \mapsto \mathcal{F} \qquad X \mapsto X_d$$
$$P \mapsto R \qquad Z \mapsto Z_d$$
$$CNOT \mapsto SUM$$

- Table math: bitwise arithmetic $\mapsto$ arithmetic mod d.

Tableau

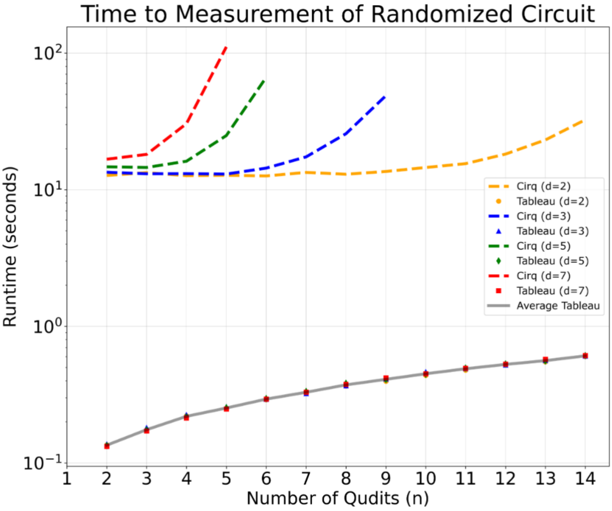
	Initial $\|00\rangle$	Apply R	Apply SUM
1	$X \otimes I$	$Z \otimes I$	$Z \otimes I$
2	$I \otimes X$	$I \otimes X$	$I \otimes X$
3	$Z \otimes I$	$X^2 \otimes I$	$X^2 \otimes X^2$
4	$I \otimes Z$	$I \otimes Z$	$Z^2 \otimes Z$

Figure: Tableau steps for the qutrit Bell state.

Our work

- Implemented a qudit tableau for in Python that is functional for any prime d.
- Implemented measurement (probabilistic evolution inherent in QC).
- Implemented automatic validation using equivalent state vector computation via Google Cirq.

Evaluation


Time to Measurement of Randomized Circuit
Runtime (seconds)
10^2
10^1
10^0
10^{-1}
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Number of Qudits (n)
Cirq (d=2)
Tableau (d=2)
Cirq (d=3)
Tableau (d=3)
Cirq (d=5)
Tableau (d=5)
Cirq (d=7)
Tableau (d=7)
Average Tableau