Simulating Quantum Computing with Stabilizers Beyond Qubits

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Abstract

Quantum computers have novel applications in cryptography and simulation of materials but currently suffer from considerable noise. Our research studies the error-correcting capabilities of qudits, the generalization of a qubit, through stabilizer codes. Such codes are powerful yet efficiently simulable via stabilizer circuit simulators on classical hardware. Until now, no qudit simulator exists despite the ubiquity of stabilizer codes and the need to benchmark different constructions under various system-scale loads. We introduce the first qudit stabilizer circuit simulator capable of simulating qudits of prime dimension.

The difficulty of qudit simulation

- Qubits have 2 complex values while qudits have $d$ complex values. These vectors represent the state $|\psi\rangle$ of the system.
- Representing $n$ qudits leads to an exponential representation.
- States $|\psi\rangle$ can be changed by multiplying the state with a $d^n \times d^n$ matrix $\mathcal{O}$ called an operator or quantum gate.

Why is simulating quantum computing hard?

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These are huge matrices. But it turns out, we can efficiently simulate a subset of states, stabilizer states.

Background on stabilizer states

- **Stabilizer states:** the set of states one can reach by applying any sequence of quantum gates from $\{H, P, CNOT\}$ to state $|0\rangle$. The gates are called Clifford gates.
- Every $n$-qubit state can be represented by a binary tableau. The celebrated Gottesman-Knill theorem demonstrates how tracking the quantum state with a tableau is only Clifford gates.
- Our work extends the binary tableau to an integer tableau for $n$-qudit stabilizer states.

Visualizing stabilizer states for a single qubit

Wait what’s the Bloch sphere?
- Visualizes transform of complex values to rotations on a sphere.
- The closer you are to $|0\rangle$ or $|1\rangle$, the more likely you are to measure them.
- Stabilizer codes essentially “round” the state $|\psi\rangle$ to a stabilizer state, the red dots on the sphere
- Clifford gates can only move you between the red dots

Example simulation of the qutrit Bell’s state

$|00\rangle, |11\rangle$, and $|22\rangle$ are stabilizer states. These vectors represent the quantum state with a tableau.

The key trick

- By tracking additional operators alongside stabilizers, we avoid matrix inversion during measurement
- We also showed that tableau operations for the generalized Pauli stabilizers become integer arithmetic modulo $d$ for prime $d > 2$, instead of bitwise operations used in $d = 2$.

Scalability of our simulator

- Tested on circuits of 100,000 random gates consisting of 40% SUM, 30% R, 30% P gates
- Performance is polynomial in qudit count compared to statevector simulation’s exponential performance
- Note that the tableau performance is independent of the qudit dimension

Future Direction

- Move from a standalone Python implementation to integrating with an existing stabilizer simulator such as Stim, QuantumClifford.jl, or even a more general quantum computing simulation platform like Qiskit and Cirq.
- Leverage the above platforms’ mature and extensive noise profiling tools for thorough error-correction benchmarking and empirical study.
- Publicize the tool as a contribution toward the growing landscape of practical qudit research from theoretical study to concrete applications.

References