Quantum computing fundamentals: State, Composition, Dynamics

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QC Algos

States Gates Protocols Algos Repetiturn

Algos
- DJ
- Simons
- BV
- Shors
- Grovers

QECC 2026

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Postulates of quantum mechanics

QIS/QIP → QM

discrete state   Continuous state

discrete time   continuous time

1. State space
2. Composite systems
3. Evolution
4. Quantum measurement { measurement. / noise

1, 2, and 3 are linear and describe closed quantum systems. 4 is nonlinear and describes open quantum systems.

closed.

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Exercise: The Qubit's Gambit

► What is the qubit state represented by the geographical location of Old Queens?

Quantum postulate 1: State space

The position or momentum of a physical system is described as a wavefunction

▶ Assuming continuous state space:

$$|\psi\rangle = \int_{-\infty}^{\infty} \psi(x) \, |x\rangle \, dx$$

$$|x\rangle$$

are orthonormal

$$\psi(x) \in \mathbb{C}$$

▶ Assuming discrete state space:

$$|\psi\rangle = \sum_{i=0}^{\infty} \psi(x_i) \, |x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

Quantum postulate 1: State space

The position or momentum of a physical system is described as a wavefunction

▶ Assuming discrete state space:

$$|\psi\rangle = \sum_{i=0}^{\infty} \psi(x_i) |x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

▶ Assuming discrete binarized state space:

$$|\psi\rangle = \sum_{i=0}^{1} \psi(x_i) |x_i\rangle$$

$$\psi(x) \in \mathbb{C}$$

$$|\psi\rangle = \psi(|0\rangle)|0\rangle +$$
$$\psi(|1\rangle)|1\rangle$$

The binary abstraction

High, low voltage

Adds resilience against noise.

Representation as a state vector

► $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$ We pronounce this "ket" 0

► $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$ We pronounce this "ket" 1

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\langle 0| = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)^t = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

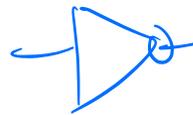$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$\langle 1|0\rangle = 0$$

The NOT gate
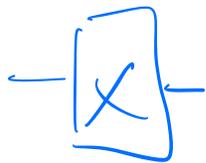
Matrix representation of NOT operator: $X = \sigma_X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

▶ $X\,|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

▶ $X\,|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

Circuit diagram representation:

bit flip / bit flip noise
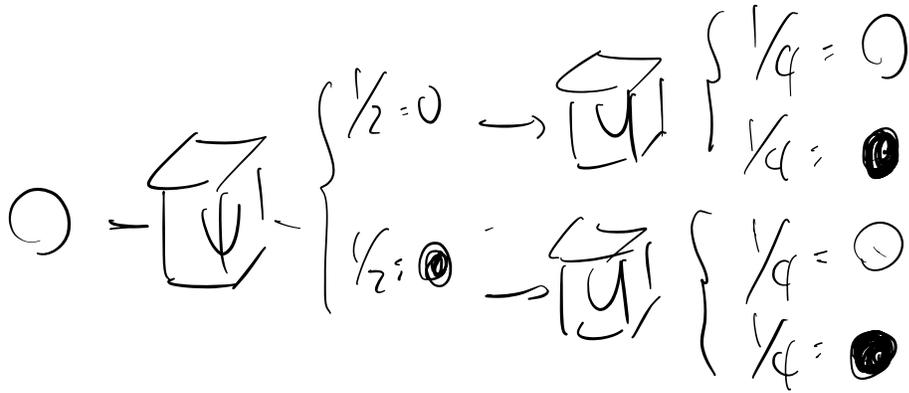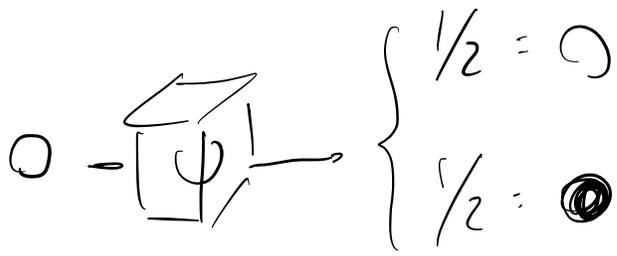
The Hadamard gate

Matrix representation of Hadamard operator: $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$

▶ $H \ket{0} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \ket{0} + \frac{1}{\sqrt{2}} \ket{1} = \ket{+}$

▶ $H \ket{1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \ket{0} - \frac{1}{\sqrt{2}} \ket{1} = \ket{-}$

Circuit diagram representation:

$$\ket{0} - \boxed{H} - \ket{+}$$

$$\bigcirc \;-\; \boxed{\psi} \;\longrightarrow\; \begin{cases} \tfrac{1}{2} = \bigcirc \\[2mm] \tfrac{1}{2} = \bullet \end{cases}$$

$$\bigcirc \;-\; \boxed{\psi} \;-\; \begin{cases} \tfrac{1}{2} = 0 \;\longrightarrow\; \boxed{U} \begin{cases} \tfrac{1}{4} = \bigcirc \\[2mm] \tfrac{1}{4} = \bullet \end{cases} \\[6mm] \tfrac{1}{2} = \bullet \;\longrightarrow\; \boxed{U} \begin{cases} \tfrac{1}{4} = \bigcirc \\[2mm] \tfrac{1}{4} = \bullet \end{cases} \end{cases}$$

$$\bigcirc \;-\; \boxed{U\,|\,U} \;\longrightarrow\; \bigcirc$$

Interference

Amplitudes can positively and negatively interfere

▶ $HH\,|0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$
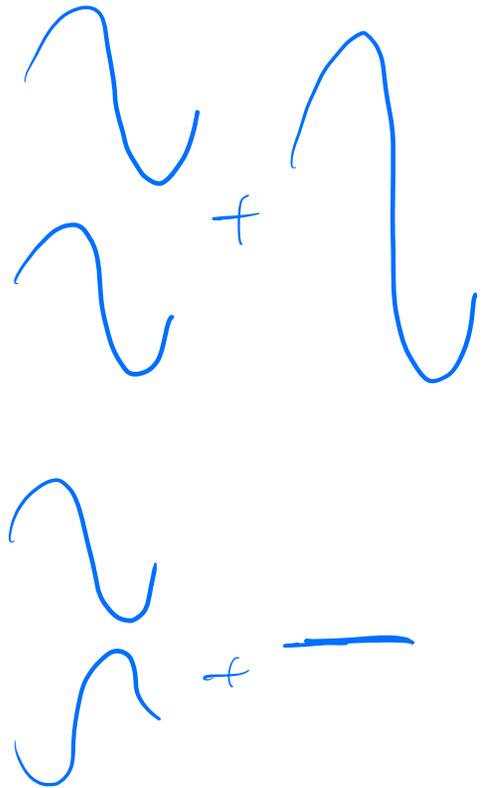
▶ $HH\,|1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} + \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

Circuit diagram representation:

$|+\rangle - [H] \longrightarrow |0\rangle$

$|-\rangle - [H] \longrightarrow |1\rangle$

Superposition

Single qubit state

▶ $\alpha \,|0\rangle + \beta \,|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

▶ Amplitudes $\alpha, \beta \in \mathbb{C}$

▶ $|\alpha|^2 + |\beta|^2 = 1$

▶ The above constraints require that qubit operators are unitary matrices.

Many physical phenomena can be in superposition and encode qubits

▶ Polarization of light in different directions

▶ Electron spins (Intel solid state qubits)

▶ Atom energy states (UMD, IonQ ion trap qubits)

▶ Quantized voltage and current (IBM, Google superconducting qubits)

If multiple discrete values are possible (e.g., atom energy states, voltage and current), we pick (bottom) two for the binary abstraction.

The phase shift gate

Pauli-Z

Matrix representation of phase shift operator: $Z = \sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

▶ $Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

$Z|+\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = |-\rangle$

▶ $Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$

$Z|-\rangle = |+\rangle$

Circuit diagram representation:

$|+\rangle - \boxed{Z} - |-\rangle$

$|-\rangle - \boxed{Z} - |+\rangle$

A simple physics experiment that classical computing cannot replicate

Algorithm

David Deutsch and Richard Jozsa. Rapid solution of problems by quantum computation. 1992.

Implementation

- ▶ Mach-Zehnder interferometer implementation.
  `https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/SinglePhotonLab/SinglePhotonLab.html`

Mathematical description of the algorithm

$$|0\rangle \xrightarrow{H} |+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{cases} \xrightarrow{I} |+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} |0\rangle \\[2em] \xrightarrow{Z} |-\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} |1\rangle \\[2em] \xrightarrow{-Z} -|-\rangle = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} -|1\rangle \\[2em] \xrightarrow{-ZZ=-I} -|+\rangle = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} -|0\rangle \end{cases}$$
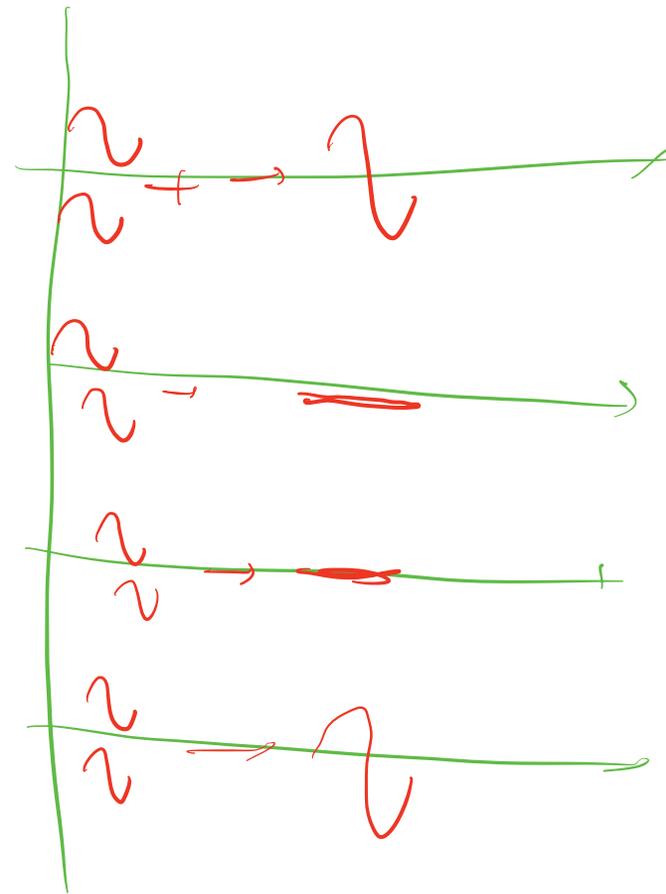
Bloch sphere

Representation of pure states of a single qubit

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

▶ θ polar angle
▶ ϕ azimuthal angle

Euler's formula

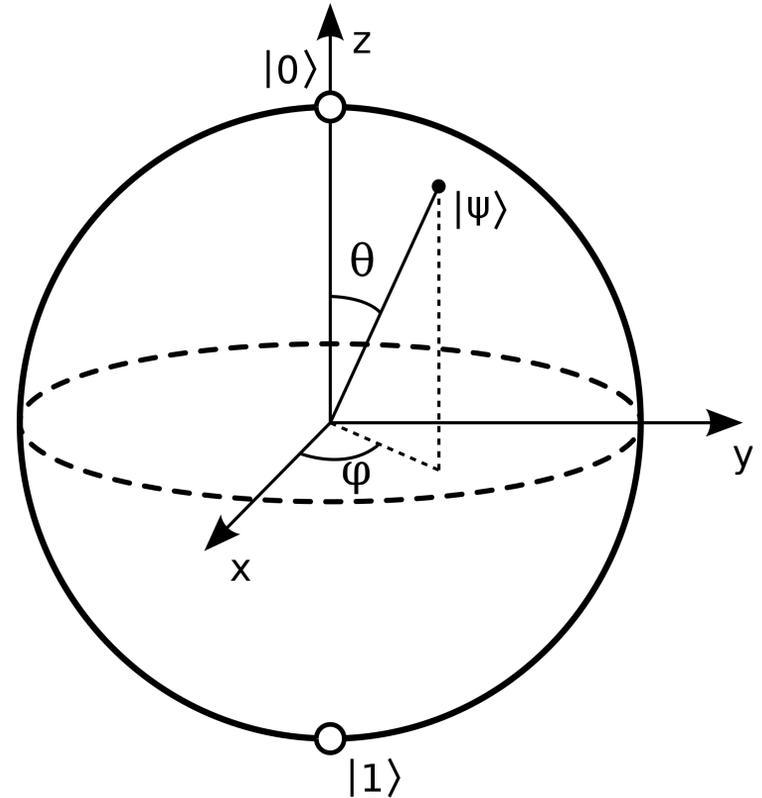$$e^{i\phi} = \cos\phi + i\sin\phi$$

Figure: Source: Wikimedia

Bloch sphere

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Important locations on the Bloch sphere

- $|+\rangle = H\,|0\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$
- $|-\rangle = H\,|1\rangle = \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle$
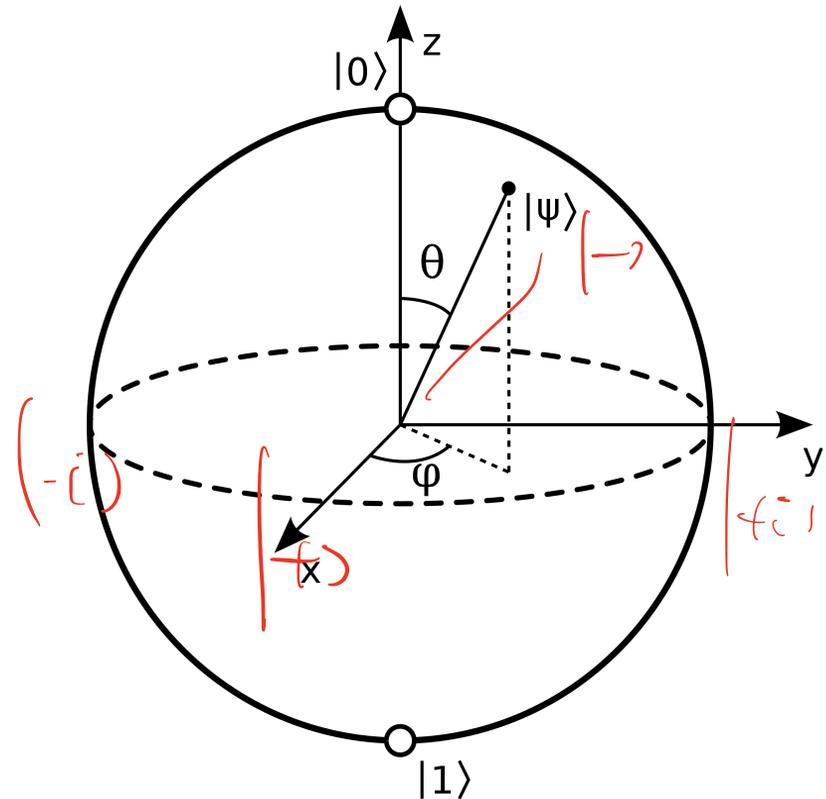

Figure: Source: Wikimedia

What is state @ +y direction, unit distance
from origin!

$$|\psi\rangle = \cos\frac{\pi/2}{2}|0\rangle + e^{i\frac{\pi}{2}}\sin\frac{\pi/2}{2}|1\rangle$$

$$= \frac{|0\rangle}{\sqrt{2}} + i\frac{|1\rangle}{\sqrt{2}} = |+i\rangle$$

$$R_{\hat{z}}(180°)|+i\rangle = Z|+i\rangle$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{bmatrix}$$

$$= |-i\rangle$$

Bloch sphere

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Rotations around the Bloch sphere

▶

$$R_x(\theta) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X$$

▶

$$R_y(\theta) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y$$

▶

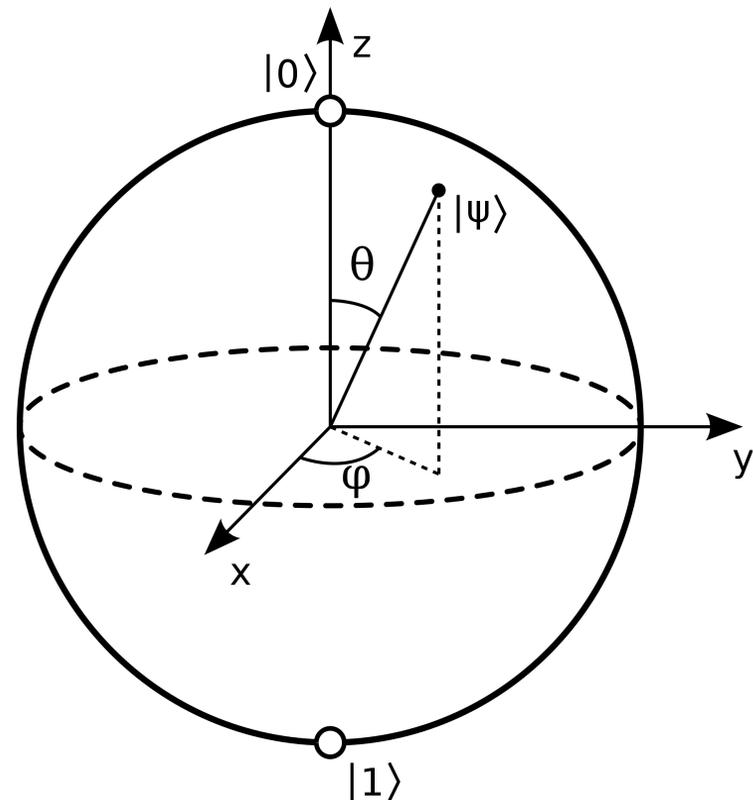$$R_z(\theta) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z$$

Figure: Source: Wikimedia

Exercise: The Qubit's Gambit

▶ Assume that the Earth is spherical.

▶ Let the North Pole be $|0\rangle$.

▶ Let $|+\rangle$ be on the prime meridian.

▶ What is the qubit state represented by the geographical location of Old Queens?

Stabilizer Representation

Clifford Operations

Non-Clifford Operations

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Quantum postulate 2: Composite systems

The state space of composite systems is the tensor product of state space of component systems.

Multiple qubits: the tensor product

Tensor product (also known as Kronecker product) of state vectors

$$|+\rangle \otimes |-\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \\ \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \end{bmatrix} = \frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$$

Multiple qubits: the tensor product

Tensor product of unitary matrices

$$X \otimes I \left(\tfrac{1}{\sqrt{2}} \ket{01} + \tfrac{1}{\sqrt{2}} \ket{11} \right) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} =$$

$$\begin{bmatrix} 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \tfrac{1}{\sqrt{2}} \ket{01} + \tfrac{1}{\sqrt{2}} \ket{11}$$

Circuit diagram representation:

Multiple qubits: the tensor product

Tensor product of state vectors

$$X\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle\right) \otimes I\left|1\right\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\left|01\right\rangle + \frac{1}{\sqrt{2}}\left|11\right\rangle$$

Circuit diagram representation:

Multiple qubits: the tensor product

Exercise: proof by induction about the Hadamard transform

Show that $|+\rangle^{\otimes n} = \frac{1}{2^{n/2}} \sum_{m=0}^{2^n-1} |m\rangle$

Exercise: Superposition of computational basis states

1. Show that
$$(H \otimes H)|00\rangle = |+\rangle \otimes |+\rangle$$

2. Show that
$$|+\rangle^{\otimes n} = \frac{1}{2^{n/2}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{2^n \times 1}$$

3. Draw a quantum circuit that yields the above state for n qubits.

Entangled states: Bell state circuit

Bell state circuit

$$|00\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|00\rangle + |10\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|00\rangle + |11\rangle \Big) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |\Phi^+\rangle$$

Can $|\Phi^+\rangle$ be treated as the tensor product (composition) of two individual qubits?

Prove that the Bell state cannot be factored into two single-qubit states

Bell state circuit

$$|00\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|00\rangle + |10\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|00\rangle + |11\rangle \Big) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |\Phi^+\rangle$$

Can $|\Phi^+\rangle$ be treated as the tensor product (composition) of two individual qubits?
No.

Bell states form an orthogonal basis set

1. $|00\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|00\rangle + |10\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|00\rangle + |11\rangle \Big) = |\Phi^+\rangle$

2. $|01\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|01\rangle + |11\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|01\rangle + |10\rangle \Big) = |\Psi^+\rangle$

3. $|10\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|00\rangle - |10\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|00\rangle - |11\rangle \Big) = |\Phi^-\rangle$

4. $|11\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}} \Big(|01\rangle - |11\rangle \Big) \xrightarrow{CNOT} \frac{1}{\sqrt{2}} \Big(|01\rangle - |10\rangle \Big) = |\Psi^-\rangle$

No-cloning theorem

There is no way to duplicate an arbitrary quantum state

Suppose a cloning operation U_c exists. Then:

▶

$$U_c(|\phi\rangle \otimes |\omega\rangle) = |\phi\rangle \otimes |\phi\rangle \, ,$$

$$U_c(|\psi\rangle \otimes |\omega\rangle) = |\psi\rangle \otimes |\psi\rangle \, ,$$

for arbitrary states $|\phi\rangle , |\psi\rangle$ we wish to copy.

▶ The overlap of the initial states is:

$$\langle\phi| \otimes \langle\omega| |\psi\rangle \otimes |\omega\rangle = \langle\phi| |\psi\rangle \cdot \langle\omega| |\omega\rangle = \langle\phi| |\psi\rangle$$

No-cloning theorem

There is no way to duplicate an arbitrary quantum state

Suppose a cloning operation U_c exists. Then:

▶
$$U_c(|\phi\rangle \otimes |\omega\rangle) = |\phi\rangle \otimes |\phi\rangle \,,$$

$$U_c(|\psi\rangle \otimes |\omega\rangle) = |\psi\rangle \otimes |\psi\rangle \,,$$

for arbitrary states $|\phi\rangle, |\psi\rangle$ we wish to copy.

▶ The overlap of the final states is:

$$\langle\phi| \otimes \langle\phi| |\psi\rangle \otimes |\psi\rangle = \langle\phi| |\psi\rangle \cdot \langle\phi| |\psi\rangle = (\langle\phi| |\psi\rangle)^2$$

▶ The overlap of the final states is also:

$$\langle\phi| \otimes \langle\phi| |\psi\rangle \otimes |\psi\rangle = \langle\phi| \otimes \langle\omega| U^\dagger U |\psi\rangle \otimes |\omega\rangle = \langle\phi| \otimes \langle\omega| |\psi\rangle \otimes |\omega\rangle = \langle\phi| |\psi\rangle$$

▶ $(\langle\phi| |\psi\rangle)^2 = \langle\phi| |\psi\rangle$, so $\langle\phi| |\psi\rangle = 0$, or $\langle\phi| |\psi\rangle = 1$, $|\phi\rangle$ and $|\psi\rangle$ cannot be arbitrary states as claimed.

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Quantum postulate 3: Evolution

The time evolution of a state follows the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle$$

▶ Comes from the conservation of total energy in the closed system, one of the observables from the system state.

▶ Itself reflects a time-invariance.

$$\frac{\partial}{\partial t} |\psi(t)\rangle = \frac{-iH}{\hbar} |\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{\frac{-iH}{\hbar}} |\psi(t)\rangle$$

Quantum postulate 3: Evolution

The evolution of a closed quantum system is a unitary transformation.

$$|\psi(t = t_1)\rangle = U |\psi(t = t_0)\rangle$$

▶ $|\psi_1\rangle = U |\psi_0\rangle$

▶ In a closed quantum system, $\langle\psi_1| |\psi_1\rangle = \langle\psi_0| U^\dagger U |\psi_0\rangle = \langle\psi_0| |\psi_0\rangle = 1$

▶ $U^\dagger U = I, U^\dagger = U^{-1}$; Such matrices U are unitary

Quantum postulate 3: Evolution

From unitary transformations we can show Hamiltonians in closed quantum systems must be hermitian

- $U \ket{\psi} = e^{\frac{-iH}{\hbar}} \ket{\psi}$

- $U^\dagger \ket{\psi} = e^{\frac{-(iH)^\dagger}{\hbar}} \ket{\psi}$

- $U^\dagger \ket{\psi} = U^{-1} \ket{\psi} = e^{\frac{iH}{\hbar}} \ket{\psi}$

- $(iH)^\dagger = -iH$, $A = iH$; such matrices A are called anti-Hermitian a.k.a. skew-Hermitian

- If iH is skew-Hermitian, H is Hermitian a.k.a. self-adjoint: $H^\dagger = H$