Basic quantum algorithms: Deutsch / Deutsch-Jozsa

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February 24, 2026

Semester Plan

Tuesday 3/24

Basic Algorithms

Q Error Correction

Everything you can do with 4 qubits

b/w
midterm 1
midterm 2

Today's Plan

Review of stab tableau w.r.t. Bell's circuit measurement

Deutsch

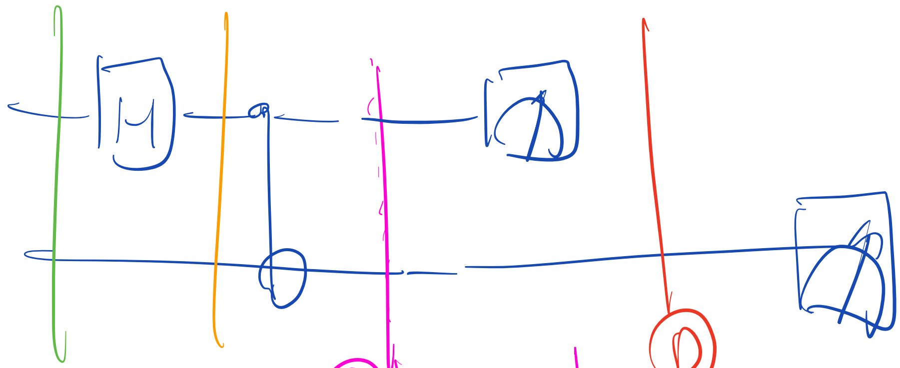
- ↳ statevector
- ↳ stab view

D-J

HW Exercises

558 Projects.

Bell State Stabilizer View



A B C D

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

A

	z_0	z_1	x_0	x_1	r		
S_0	1	0	0	0	0	:	$Z \otimes I$
S_1	0	1	0	0	0	:	$I \otimes Z$

$$(Z \otimes I)|00\rangle = |00\rangle$$

$$(I \otimes Z)|00\rangle = |00\rangle$$

$$(Z \otimes Z)|00\rangle = |00\rangle$$

$$(I \otimes I)|00\rangle = |00\rangle$$

(B)

	z_0	z_1	x_0	x_1	r	
S_0	0	0	1	0	0	XI
S_1	0	1	0	0	0	IZ

$$\text{Stab}\left\{\begin{matrix} X\otimes I \\ Z\otimes Z \\ X\otimes Z \\ I\otimes I \end{matrix}\right| +0\rangle = |+0\rangle$$

$$\rho = \frac{XI + IZ + XZ + II}{2} = |+0\rangle\langle +0|$$

(C)

	z_0	z_1	x_0	x_1	r	
S_0	0	0	1	1	0	XX
S_1	1	1	0	0	0	ZZ

$$\text{Cnot}(XI)\text{Cnot}^{\dagger} = XX$$
$$\text{Cnot}(IZ)\text{Cnot}^{\dagger} = ZZ$$

$$\text{Stabs}\left\{\begin{matrix} XX \\ ZZ \\ -YY \\ II \end{matrix}\right| \Phi^{+}\rangle = |\Phi^{+}\rangle$$

$$|\Phi^{+}\rangle\langle\Phi^{+}| = \frac{II + XX - YY + ZZ}{2}$$

(D) measure Z_0

	z_0	z_1	x_0	x_1	r	
S_0	1	0	0	0	0/1	$+ZI$ / $-ZI$
S_1	1	1	0	0	0	ZZ

top qubit meas "0"

ZI, ZZ, IZ, II

$$\rho = \frac{II + IZ + ZI + ZZ}{4}$$

$$= \frac{(I+Z)}{2} \otimes \frac{(I+Z)}{2}$$

$$= |0\rangle\langle 0| \otimes |0\rangle\langle 0|$$

top qubit meas "1"

$-ZI$, ZZ, $-IZ$, II

$$\rho = \frac{II - IZ - ZI + ZZ}{4}$$

$$= \frac{(I-Z)}{2} \otimes \frac{(I-Z)}{2}$$

$$= |1\rangle\langle 1| \otimes |1\rangle\langle 1|$$

Classical Algos.

- divide conquer
- search sort.
- greedy
- NP - heuristics
- Monte Carlo / randomize
- Dynamic Prog.

Quantum Algorithms.

- Phase kickback (Shor's Algo)
- Amplitude Amplification (Grover's Algo)
- Optimization
- Variational Eigensolvers

Promise algorithms vs. unstructured search

'70s

↓

Quantum algorithms offer exponential speedup in "promise" problems

'92

A progression of related algorithms:

1. Deutsch's -
2. Deutsch-Jozsa
3. Bernstein-Vazirani -
4. Simon's
5. Shor's -

~'96

property of the f(x)

Friday 3/27

Deutsch Jozsa

↳ Statevector Derivation

↳ Stabilizer Derivation

↳ $n>1$ case

Bernstein Vazirani

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Deutsch-Jozsa algorithm: simplest quantum algorithm showing advantage vs. classical

A Heist

- You break into a bank vault. The bank vault has 2^n bars. Three possibilities: all are gold, half are gold and half are fake, or all are fake.
- Even if you steal just one gold bar, it is enough to fund your escape from the country, forever evading law enforcement.
- You do not want to risk stealing from a bank vault with only fake bars.
- You have access to an oracle $f(x)$ that tells you if gold bar x is real.
- Using the oracle sounds the alarm, so you only get to use it once.

Deutsch-Jozsa algorithm: simplest quantum algorithm showing advantage vs. classical

More formal description

- The 2^n bars are either fake or gold. $f : \{0,1\}^n \to \{0,1\}$.
- Three possibilities:
  1. All are fake. f is constant. $f(x) = 0$ for all $x \in \{0,1\}^n$.
  2. All are gold. f is constant. $f(x) = 1$ for all $x \in \{0,1\}^n$.
  3. Half fake half gold. f is balanced.

     $$\Big|\{x \in \{0,1\}^n : f(x) = 0\}\Big| = \Big|\{x \in \{0,1\}^n : f(x) = 1\}\Big| = 2^{n-1}$$
- The oracle U works as follows: $U|c\rangle|t\rangle = |c\rangle|t \oplus f(c)\rangle$
- Try deciding if f is constant or balanced using oracle U only once.

What is in the oracle

For $n = 1$, four possibilities

	f_0	f_1	f_2	f_3
f(0)	0	0	1	1
f(1)	0	1	0	1
	f is constant 0	f is balanced	f is balanced	f is constant 1

Demonstration of Deutsch-Jozsa for the $n = 1$ case

Output of circuit is $c = 0$ iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle \otimes |1\rangle = |0\rangle |1\rangle = |01\rangle$
2. After first set of Hadamards: $H \otimes H\Big(|0\rangle \otimes |1\rangle \Big) = H|0\rangle \otimes H|1\rangle = |+\rangle \otimes |-\rangle =$

$$\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

From here, let's take an aside via matrix-vector multiplication to build intuition with interference and phase kickback.

Demonstration of Deutsch-Jozsa for the $n = 1$ case

Output of circuit is $c = 0$ iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle \otimes |1\rangle = |0\rangle\,|1\rangle = |01\rangle$
2. After first set of Hadamards: $H \otimes H\Big(\,|0\rangle \otimes |1\rangle\,\Big) = |+\rangle\,|-\rangle =$
$$\left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right)\left(\frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle\right) = \frac{1}{2}\Big(\,|0\rangle\,(\,|0\rangle - |1\rangle\,) + |1\rangle\,(\,|0\rangle - |1\rangle\,)\Big)$$
3. After applying oracle U:
$$U\frac{1}{2}\Big(\,|0\rangle\,(\,|0\rangle - |1\rangle\,) + |1\rangle\,(\,|0\rangle - |1\rangle\,)\Big) = \frac{1}{2}\Big(\,|0\rangle\,(\,|f(0) \oplus 0\rangle - |f(0) \oplus 1\rangle\,) +$$
$$|1\rangle\,(\,|f(1) \oplus 0\rangle - |f(1) \oplus 1\rangle\,)\Big) = \frac{1}{2}\Big(\,|0\rangle\,(\,|f(0)\rangle - |f(\bar{0})\rangle\,) + |1\rangle\,(\,|f(1)\rangle - |f(\bar{1})\rangle\,)\Big)$$

Demonstration of Deutsch-Jozsa for the $n = 1$ case

Output of circuit is $c = 0$ iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle \otimes |1\rangle = |0\rangle|1\rangle = |01\rangle$
2. After first set of Hadamards: $\frac{1}{2}\Big(|0\rangle(|0\rangle - |1\rangle) + |1\rangle(|0\rangle - |1\rangle)\Big)$
3. After applying oracle U: $U\frac{1}{2}\Big(|0\rangle(|0\rangle - |1\rangle) + |1\rangle(|0\rangle - |1\rangle)\Big) =$
   $\frac{1}{2}\Big(|0\rangle\big(|f(0)\rangle - |\bar{f(0)}\rangle\big) + |1\rangle\big(|f(1)\rangle - |\bar{f(1)}\rangle\big)\Big)$
4. This last expression can be factored depending on f:
   $U\frac{1}{2}\Big(|0\rangle(|0\rangle - |1\rangle) + |1\rangle(|0\rangle - |1\rangle)\Big) =$
   $\begin{cases} \frac{1}{2}\big(|0\rangle + |1\rangle\big)\big(|f(0)\rangle - |\bar{f(0)}\rangle\big) \text{ if } f(0) = f(1) \\ \frac{1}{2}\big(|0\rangle - |1\rangle\big)\big(|f(0)\rangle - |\bar{f(0)}\rangle\big) \text{ if } f(0) \neq f(1) \end{cases} = \begin{cases} |+\rangle|-\rangle \text{ if } f(0) = f(1) \\ |-\rangle|-\rangle \text{ if } f(0) \neq f(1) \end{cases}$

The trick where oracle's output on $|t\rangle$ affects phase of $|c\rangle$ is called phase kickback.

Demonstration of Deutsch-Jozsa for the $n = 1$ case

Output of circuit is $c = 0$ iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle \otimes |1\rangle = |0\rangle\,|1\rangle = |01\rangle$
2. After first set of Hadamards: $\frac{1}{2}\Big(|0\rangle\,(|0\rangle - |1\rangle) + |1\rangle\,(|0\rangle - |1\rangle)\Big)$
3. After applying oracle U:
$$U\frac{1}{2}\Big(|0\rangle\,(|0\rangle - |1\rangle) + |1\rangle\,(|0\rangle - |1\rangle)\Big) = \begin{cases} |+\rangle\,|-\rangle & \text{if } f(0) = f(1) \\ |-\rangle\,|-\rangle & \text{if } f(0) \neq f(1) \end{cases}$$
4. After applying second H on top qubit:
$$\begin{cases} H \otimes I\big(\,|+\rangle\,|-\rangle\,\big) = |0\rangle\,|-\rangle & \text{if } f(0) = f(1) \\ H \otimes I\big(\,|-\rangle\,|-\rangle\,\big) = |1\rangle\,|-\rangle & \text{if } f(0) \neq f(1) \end{cases}$$

Deutsch-Jozsa programs and systems

Algorithm

David Deutsch and Richard Jozsa. Rapid solution of problems by quantum computation. 1992.

Programs

Google Cirq programming example.

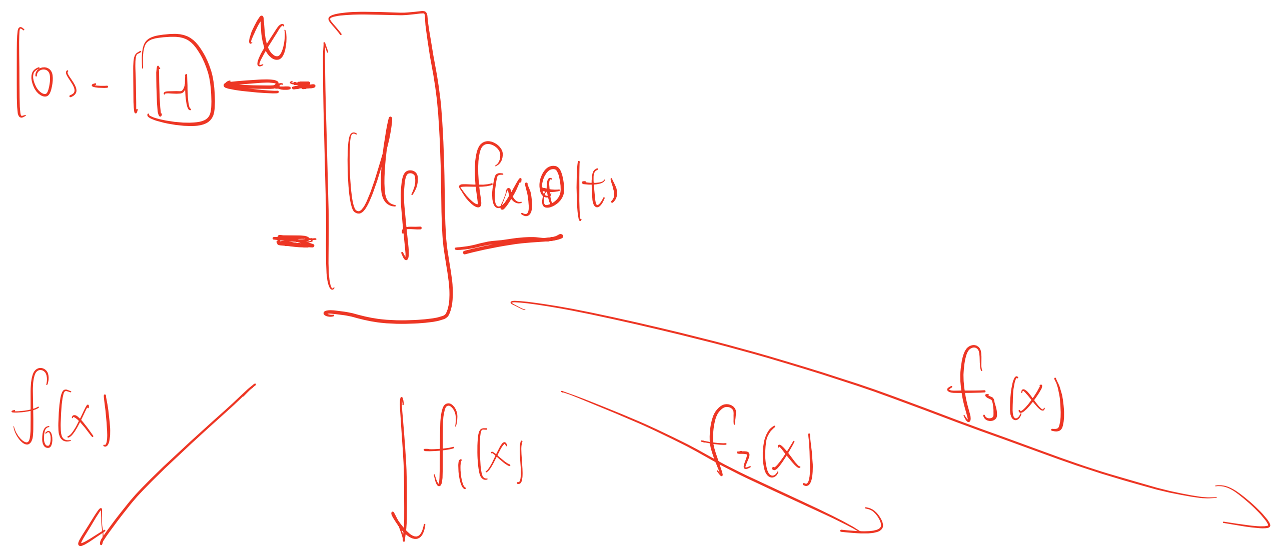
Implementation

- Mach-Zehnder interferometer implementation. `https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/SinglePhotonLab/SinglePhotonLab.html`
- Ion trap implementation. Gulde et al. Implementation of the Deutsch–Jozsa algorithm on an ion-trap quantum computer. Letters to Nature. 2003.

Mach-Zehnder interferometer implementation of Deutsch's algorithm

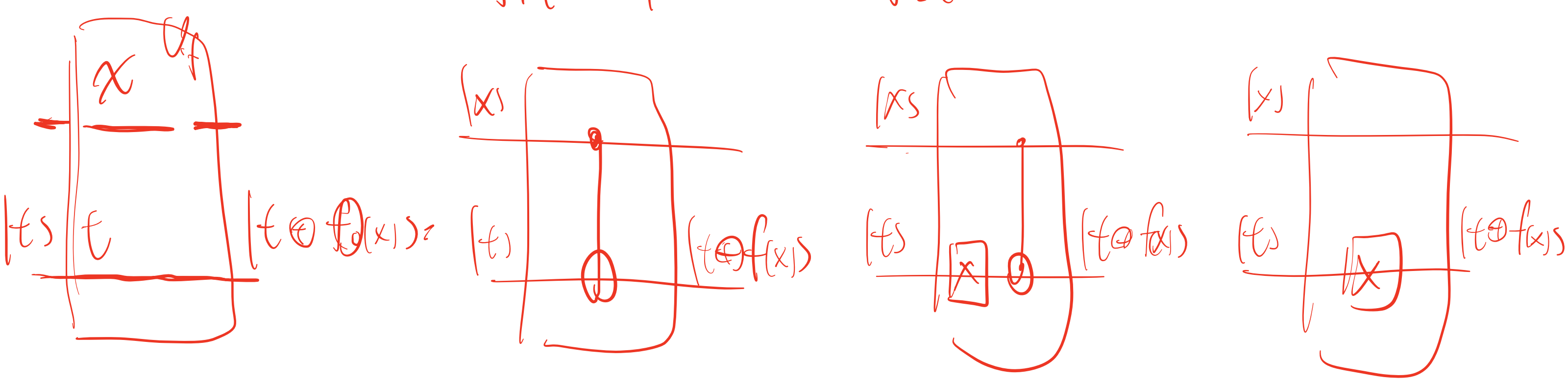
$$|0\rangle \xrightarrow{H} |+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{cases} \xrightarrow{I} |+\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} |0\rangle \\ \xrightarrow{Z} |-\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} |1\rangle \\ \xrightarrow{-Z} -|-\rangle = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} -|1\rangle \\ \xrightarrow{-ZZ=-I} -|+\rangle = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} & \xrightarrow{H} -|0\rangle \end{cases}$$

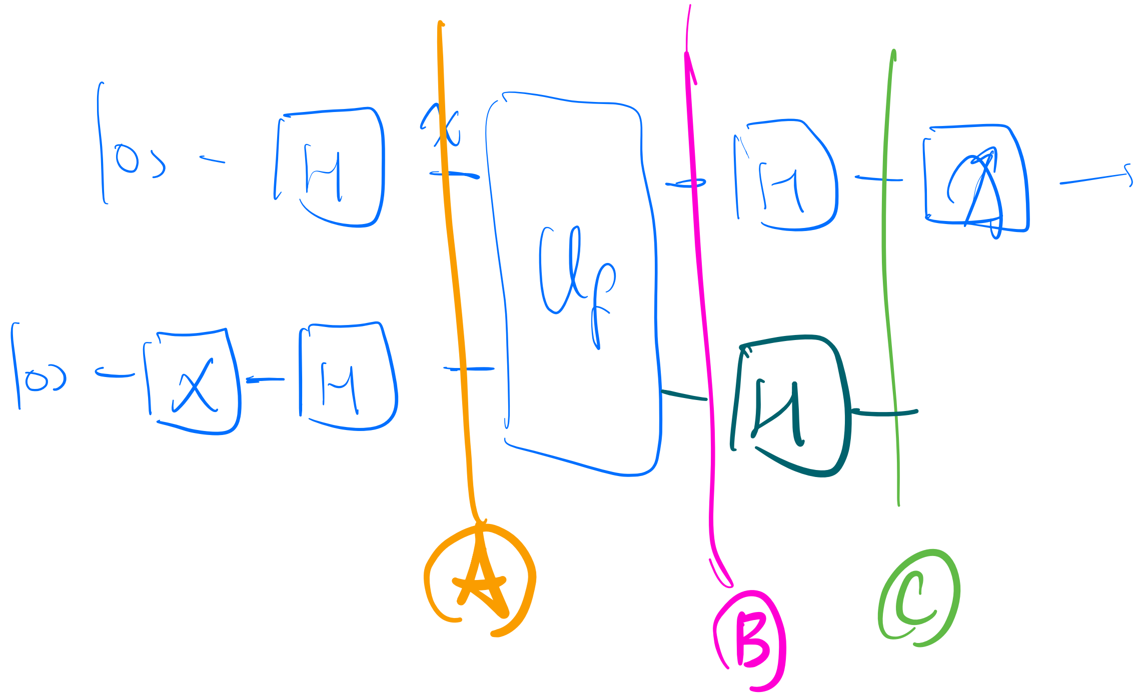
Deutsch Statevector View



$f_0(0) = 0$ $f_1(0) = 0$ $f_2(0) = 1$ $f_3(0) = 1$

$f_0(1) = 0$ $f_1(1) = 1$ $f_2(1)\ 0$ $f_3(1) = 1$





$$\begin{cases} 0: f \text{ is const} \\ 1: f \text{ is bal} \end{cases}$$

A $|+\rangle|-\rangle$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

B_{f_2} $$\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \xrightarrow{I \otimes X} \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \xrightarrow{CNOT} \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

C_{f_2} $$\frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = -|-\rangle \otimes |-\rangle \xrightarrow{H \otimes H} -|1\rangle \otimes |1\rangle$$

B_{f_3} $$\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \xrightarrow{I \otimes X} \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

C_{f_3} $$\frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = -|+\rangle \otimes |-\rangle \xrightarrow{H \otimes H} -|0\rangle \otimes |1\rangle$$

Deutsch Algo Stabilizer View

A

	z_0	z_1	x_0	x_1	r
S_0	1	0	0	0	0
S_1	0	1	0	0	0

$Z|0\rangle = |0\rangle$

$XZX^\dagger = -Z$

$I \otimes X$

	z_0	z_1	x_0	x_1	r
S_0	1	0	0	0	0
S_1	0	1	0	0	1

$$\rho = |01\rangle\langle 01| = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$Z \otimes I|01\rangle = |01\rangle$

$-I \otimes Z|01\rangle = |01\rangle$

$$\rho = \frac{II - IZ + ZI - ZZ}{4}$$

$$= \begin{bmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{bmatrix} - \begin{bmatrix}1&&&\\&-1&&\\&&1&\\&&&-1\end{bmatrix} + \begin{bmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{bmatrix} - \begin{bmatrix}1&&&\\&-1&&\\&&-1&\\&&&1\end{bmatrix}$$

$H \otimes H$

	z_0	z_1	x_0	x_1	r
S_0	0	0	1	0	0
S_1	0	0	0	1	1

B_{f_0} / B_{f_3}

$I \otimes X$

$X X X^{\dagger} = X$

	z_0	z_1	x_0	x_1	r
S_0	0	0	1	0	0
S_1	0	0	0	1	1

B_{f_1} B_{f_2}

Cnot

	z_0	z_1	x_0	x_1	r
S_0	0	0	1	1	0
S_1	0	0	0	1	1

C_{const} $H \otimes I$

	z_0	z_1	x_0	x_1	r
S_0	1	0	0	0	0
S_1	0	0	0	1	1

$\downarrow I \otimes H$

	z_0	z_1	x_0	x_1	r
S_0	1	0	0	0	0
S_1	0	1	0	0	1

$ZI, -IZ, -ZZ, II$

$= \frac{I+Z}{2} \otimes \frac{I-Z}{2}$

$|0\rangle \otimes |1\rangle$

C_{bal} $H \otimes I$

	z_0	z_1	x_0	x_1	r
S_0	1	0	0	1	0
S_1	0	0	0	1	1

$\downarrow I \otimes H$

	z_0	z_1	x_0	x_1	r
S_0	1	1	0	0	0
S_1	0	1	0	0	1

$ZZ, -IZ, -ZI, II$

$= \frac{I-Z}{2} \otimes \frac{I-Z}{2}$

$|1\rangle \otimes |1\rangle$

$$|0\rangle\langle 0| = \frac{I+Z}{2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \frac{\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}}{2} + \frac{\begin{bmatrix} 1 & \\ & -1 \end{bmatrix}}{2}$$

$$|1\rangle\langle 1| = \frac{I-Z}{2}$$

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	what is the f(x)	classical queries:	quantum computer:
Deutsch Jozsa	$\lvert t\rangle \to \lvert t \oplus f(x)\rangle$ f(x) const or balanced	worst case $\frac{2^n}{2}+1$ propabilistic	1
Bernstein Vazirani	$f(x) = x \cdot s$ find s	n, but, cannot approximate	1
Simon's			
Shor's			

Deutsch Jozsa Oracle

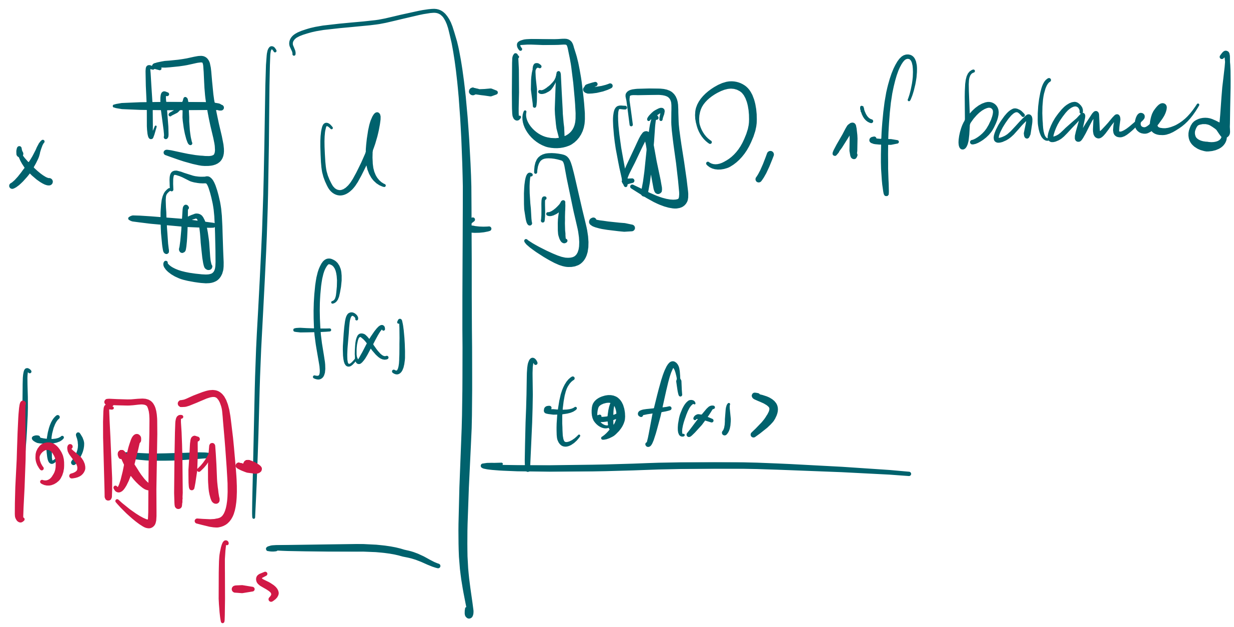
$f(x) \in \{0,1\}$

f(x)	f_0	f_3	f_5	f_6	f_9	f_{10}	f_{12}	f_{15}
x: 00	0	0	0	0	1	1	1	1
x: 01	0	0	1	1	0	0	1	1
x: 10	0	1	0	1	0	1	0	1
x: 11	0	1	1	0	1	0	0	1

$\{0,1\}^n$

$\underbrace{f_3 \; f_5 \; f_6 \; f_9 \; f_{10} \; f_{12}}_{4C2}$

Deutsch Jozsa Circuit



key things to see.

1. notation
2. factoring equations
3. constructive/destructive interference

Deutsch-Jozsa algorithm: Deutsch's algorithm for the $n > 1$ case

The state after the first set of Hadamards

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle^{\otimes n} \otimes |1\rangle = |0 \ldots 0\rangle |1\rangle = |0 \ldots 01\rangle$
2. After first set of Hadamards: $|+\rangle^{\otimes n} \otimes |-\rangle = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes |-\rangle$

Deutsch's algorithm: Deutsch-Jozsa for the $n = 1$ case

The state after applying oracle U

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle^{\otimes n} \otimes |1\rangle = |0\ldots0\rangle\,|1\rangle = |0\ldots01\rangle$
2. After first set of Hadamards: $|+\rangle^{\otimes n} \otimes |-\rangle = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes |-\rangle$
3. After applying oracle U:

$$
\begin{aligned}
U\Big(|+\rangle^{\otimes n} \otimes |-\rangle\Big) &= \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes \left(\frac{|f(c)\rangle - |\overline{f(c)}\rangle}{\sqrt{2}}\right) \\
&= \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)}\,|c\rangle \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)
\end{aligned}
$$

Lemma: the Hadamard transform

$H^{\otimes n}|c\rangle = \frac{1}{2^{n/2}}\sum_{m=0}^{2^n-1}(-1)^{c\cdot m}|m\rangle$

- $$\begin{aligned} &H^{\otimes n}|c\rangle \\ &= H|c_0\rangle \otimes H|c_1\rangle \otimes \ldots \otimes H|c_{n-1}\rangle \\ &= \frac{1}{\sqrt{2}}\Big(|0\rangle + (-1)^{c_0}|1\rangle\Big) \otimes \frac{1}{\sqrt{2}}\Big(|0\rangle + (-1)^{c_1}|1\rangle\Big) \otimes \ldots \otimes \frac{1}{\sqrt{2}}\Big(|0\rangle + (-1)^{c_{n-1}}|1\rangle\Big) \\ &= \frac{1}{2^{n/2}}\sum_{m=0}^{2^n-1}(-1)^{c_0m_0+c_1m_1+\ldots+c_{n-1}m_{n-1} \mod 2}|m\rangle \end{aligned}$$
- Try it out for $n=1$: $H^{\otimes 1}|c\rangle = \frac{1}{2^{1/2}}\sum_{m=0}^{2^1-1}(-1)^{c\cdot m}|m\rangle =$
$\frac{1}{\sqrt{2}}(-1)^0|0\rangle + \frac{1}{\sqrt{2}}(-1)^c|1\rangle = \begin{cases} \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle & \text{if } |c\rangle = |0\rangle \\ \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle & \text{if } |c\rangle = |1\rangle \end{cases}$

Deutsch-Jozsa algorithm: Deutsch's algorithm for the $n > 1$ case

The state after applying oracle U

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle^{\otimes n} \otimes |1\rangle = |0\ldots0\rangle\,|1\rangle = |0\ldots01\rangle$
2. After first set of Hadamards: $|+\rangle^{\otimes n} \otimes |-\rangle = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes |-\rangle$
3. After applying oracle U: $U\Big(|+\rangle^{\otimes n} \otimes |-\rangle\Big) = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \Big(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\Big)$
4. After final set of Hadamards:

$$
\begin{aligned}
&(H^{\otimes n} \otimes I)\left(\frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right) \\
&= \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} \left(\frac{1}{2^{n/2}} \sum_{m=0}^{2^n-1} (-1)^{c\cdot m} |m\rangle\right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\
&= \frac{1}{2^n} \sum_{c=0}^{2^n-1} \sum_{m=0}^{2^n-1} (-1)^{f(c)+c\cdot m} |m\rangle \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)
\end{aligned}
$$

Deutsch-Jozsa algorithm: Deutsch's algorithm for the $n > 1$ case

Output of circuit is 0 iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle^{\otimes n} \otimes |1\rangle = |0\ldots0\rangle\,|1\rangle = |0\ldots01\rangle$
2. After first set of Hadamards: $|+\rangle^{\otimes n} \otimes |-\rangle = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes |-\rangle$
3. After applying oracle U: $U\Big(\,|+\rangle^{\otimes n} \otimes |-\rangle\,\Big) = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$
4. After final set of Hadamards: $(H^{\otimes n} \otimes I)\left(\frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right) =$
   $\frac{1}{2^n} \sum_{c=0}^{2^n-1} \sum_{m=0}^{2^n-1} (-1)^{f(c)+c\cdot m} |m\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$
5. Amplitude of upper register being $|m\rangle = |0\rangle$:

$$\frac{1}{2^n} \sum_{c=0}^{2^n-1} (-1)^{f(c)}$$

Deutsch-Jozsa algorithm: Deutsch's algorithm for the $n > 1$ case

Output of circuit is 0 iff f is constant

1. Initial state: $|c\rangle \otimes |t\rangle = |0\rangle^{\otimes n} \otimes |1\rangle = |0\ldots0\rangle\,|1\rangle = |0\ldots01\rangle$
2. After first set of Hadamards: $|+\rangle^{\otimes n} \otimes |-\rangle = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} |c\rangle \otimes |-\rangle$
3. After applying oracle U: $U\Big(|+\rangle^{\otimes n} \otimes |-\rangle\Big) = \frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$
4. After final set of Hadamards: $(H^{\otimes n} \otimes I)\left(\frac{1}{2^{n/2}} \sum_{c=0}^{2^n-1} (-1)^{f(c)} |c\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right) =$ $\frac{1}{2^n} \sum_{c=0}^{2^n-1} \sum_{m=0}^{2^n-1} (-1)^{f(c)+c\cdot m} |m\rangle \otimes \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$
5. Amplitude of upper register being $|m\rangle = |0\rangle$: $\frac{1}{2^n} \sum_{c=0}^{2^n-1} (-1)^{f(c)}$
6. Probability of measuring upper register to get $m = 0$:

$$\left|\frac{1}{2^n} \sum_{c=0}^{2^n-1} (-1)^{f(c)}\right|^2 = \begin{cases} \left|(-1)^{f(c)}\right|^2 = 1 & \text{if } f \text{ is constant} \\ 0 & \text{if } f \text{ is balanced} \end{cases}$$

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Exercises for the scoreboard:

New meta after Midterm 1

Score calculation formula is now $\frac{\text{multiplier} * \text{upvotes}}{\sqrt{\text{team size}}}$

Multiplier 4 problems

Implement one of these protocols or algorithms in a Python notebook, using a library such as Qiskit or Cirq, shared using Google Collaboratory, following the assignment directions:

1. Quantum dense coding
2. Quantum teleportation
3. Deutsch's algorithm, defined on two qubits
4. Deutsch-Jozsa algorithm, defined on three qubits
5. Bernstein-Vazirani algorithm