A periodic table of quantum error correction codes

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NISQ vs. FTQC

noisy intermediate scale quantum

fault-tolerant QC

quantum error correcting codes (QECC)

10, 100, 1000 qubits

NISQ

- past ~10 years

  random sampling

  optimization

  chemistry

FTQC

- 1M qubits

  phys qubits ~0.1% error

  ~1% error

  logical qubits 1/18 error

  next ? years

Quantum error correction codes: theory and practical systems

Theory cares about

- Code taxonomy
- Code parameters
- Code specification

Practical systems need

- Code words
- Encoding circuit, and its compilation
- Logical operators, and their compilation
- Error syndrome parity check circuit, and its compilation
- Decoding algorithm

What this talk is about

- A tutorial on quantum Reed-Muller codes, a taxonomy where the existence of logical operators is the most clear
- A geometric approach to establishing what operators are available, and its impact on compilation
- Stabilizers and codewords of quantum Reed-Muller codes, and their potential as symmetry-preserving quantum ansatzes.
- A correct generalization of the geometric view of quantum Reed-Muller codes to qudits.

A periodic table of classical Reed-Muller codes

1

$RM(-1,0)$ $[1,0,\infty]$ | $RM(0,0)$ $[1,1,1]$

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1 1

$RM(-1,1)$ $[2,0,\infty]$ | $RM(0,1)$ $[2,1,2]$ | $RM(1,1)$ $[2,2,1]$

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1 2 1

$RM(-1,2)$ $[4,0,\infty]$ | $RM(0,2)$ $[4,1,4]$ | $RM(1,2)$ $[4,3,2]$ | $RM(2,2)$ $[4,4,1]$

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1 3 3 1

$RM(-1,3)$ $[8,0,\infty]$ | $RM(0,3)$ $[8,1,8]$ | $RM(1,3)$ $[8,4,4]$ | $RM(2,3)$ $[8,7,2]$ | $RM(3,3)$ $[8,8,1]$

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1 4 6 4 1

$RM(-1,4)$ $[16,0,\infty]$ | $RM(0,4)$ $[16,1,16]$ | $RM(1,4)$ $[16,5,8]$ | $RM(2,4)$ $[16,11,4]$ | $RM(3,4)$ $[16,15,2]$ | $RM(4,4)$ $[16,16,1]$

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1 5 10 10 5 1

$RM(-1,5)$ $[32,0,\infty]$ | $RM(0,5)$ $[32,1,32]$ | $RM(1,5)$ $[32,6,16]$ | $RM(2,5)$ $[32,16,8]$ | $RM(3,5)$ $[32,26,4]$ | $RM(4,5)$ $[32,31,2]$ | $RM(5,5)$ $[32,32,1]$

Trivial codes | Repetition codes | Punctured Hadamard codes | Self-dual codes | Extended H | Single parit | Universe c

$RM(r = 0, m = 0)$

A single classical bit

$x: \{[0], [1]\}$

$y: \{[0], [1]\}$

$y = Gx$

- Messages x
- Codewords y
- Generator matrix $G = [1]$

A $[n = 1, k = 1, d = 1]$ code

- n
- k
- d

$RM(r = 1, m = 1) \in [n = 2, k = 2, d = 1]$

Two classical bits

$y = Gx$

- Messages $x \in \{[0, 0], [0, 1], [1, 0], [1, 1]\}$
- Codewords $y \in \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$
- Generator matrix $G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A $[n = 2, k = 2, d = 1]$ code

- n
- k
- d

$RM(r=0, m=1) \in [n=2, k=1, d=2]$

$y = Gx$

- Messages $x \in \{[0], [1]\}$
- Codewords $y \in \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$
- Generator matrix $G = \begin{bmatrix} 1 \\ \hline 1 \end{bmatrix}$, the message and parity check bits

$Hy \equiv 0 \mod 2$

- Indication that no error was detected
- Parity check matrix $H = \begin{bmatrix} 1 & 1 \end{bmatrix}$

The only single parity-check, repetition code

- $G = H^\top$, so it is also self-dual

$RM(r = 1, m = 3) \in [n = 8, k = 4, d = 4]$

Usually, we start with this code

- Smallest non-repetition error correcting code
- Is an extended Hamming code
- Is a truncated Hadamard code
- Closely related $[7, 4, 3]$ saturates the Hamming bound.

Fundamental differences between ECC and QECC

No-cloning theorem

There is no way to duplicate an arbitrary quantum state. Suppose a cloning operation U_c exists. Then:

- $$U_c(|\phi\rangle \otimes |\omega\rangle) = |\phi\rangle \otimes |\phi\rangle \,,$$
  $$U_c(|\psi\rangle \otimes |\omega\rangle) = |\psi\rangle \otimes |\psi\rangle \,,$$
  for arbitrary states $|\phi\rangle, |\psi\rangle$ we wish to copy.
- The overlap of the final states is:
  $$\langle\phi| \otimes \langle\phi| \, |\psi\rangle \otimes |\psi\rangle = \langle\phi| \, |\psi\rangle \cdot \langle\phi| \, |\psi\rangle = (\langle\phi| \, |\psi\rangle)^2$$
- The overlap of the final states is also:
  $$\langle\phi| \otimes \langle\phi| \, |\psi\rangle \otimes |\psi\rangle = \langle\phi| \otimes \langle\omega| \, U^\dagger U \, |\psi\rangle \otimes |\omega\rangle = \langle\phi| \otimes \langle\omega| \, |\psi\rangle \otimes |\omega\rangle = \langle\phi| \, |\psi\rangle$$
- $(\langle\phi| \, |\psi\rangle)^2 = \langle\phi| \, |\psi\rangle$, so $\langle\phi| \, |\psi\rangle = 0$, or $\langle\phi| \, |\psi\rangle = 1$, $|\phi\rangle$ and $|\psi\rangle$ cannot be arbitrary states as claimed.

One-qubit quantum (stabilized) states

$RM(-1,0)$ $[1,0,\infty]$	1	$RM(0,0)$ $[1,1,1]$
$QRM_0(-1,-1)$ $[[1,0,1]]$ $S=\{Z\}$ codeword$=|0\rangle$ logicals$=\emptyset$		$QRM_0(0,0)$ $[[1,0,1]]$ $S=\{X\}$ codeword$=|+\rangle$ logicals$=\emptyset$
	$QRM_0(-1,0)$ $[[1,1,1]]$ $S=\{\}$ codeword$=|\psi\rangle$ logicals$=U$	

Two-qubit quantum error "codes"

$RM(-1,1)$ $[2,0,\infty]$	1	$RM(0,1)$ $[2,1,2]$	1	$RM(1,1)$ $[2,2,1]$
$QRM_1(-1,-1)$ $[[2,0,1]]$ $S=\{Z_0,Z_1\}$ codeword= $\lvert 0\rangle \lvert 0\rangle$ logicals= $\emptyset$		$QRM_1(0,0)$ $[[2,0,2]]$ $S=$ $\{Z_0Z_1, X_0X_1\}$ codeword= $\lvert\Phi^+\rangle$ logicals= $\emptyset$		$QRM_1(1,1)$ $[[2,0,1]]$ $S=\{X_0,X_1\}$ codeword= $\lvert +\rangle \lvert +\rangle$ logicals= $\emptyset$
	$QRM_1(-1,0)$ $[[2,1,1]]$ $S=\{Z_0Z_1\}$ logicals= $\{XX,Z,S,T,\dots\}$		$QRM_1(0,1)$ $[[2,1,1]]$ $S=\{X_0X_1\}$ logicals= $\{ZZ,X,\sqrt{X},\sqrt[4]{X},\dots\}$	
		$QRM_1(-1,1)$ $[[2,2,1]]$ $S=\{\}$ logicals= $\{H,Z,ZZ,SS,X,XX,\sqrt{X}\sqrt{X}\}$		

$QRM_{m=1}(q = -1, r = 0) \in [[n = 2, k = 1, d = 1]]$

A first attempt at making a QECC

- The code's stabilizer generators
- A valid choice of codewords
- Encoding circuit
- Parity check error syndrome extraction circuit

$QRM_{m=1}(q = 0, r = 1) \in [[n = 2, k = 1, d = 1]]$

A second attempt at making a QECC

- The code's stabilizer generators
- A valid choice of codewords
- Encoding circuit
- Parity check error syndrome extraction circuit

Degrees of freedom irrelevant to theory but impactful in systems

- Choice of codewords
- Encoding circuit
- Parity check circuit

Four-qubit quantum error (detection) codes

$RM(-1,2)$ $[4,0,\infty]$	1	$RM(0,2)$ $[4,1,4]$	2	$RM(1,2)$ $[4,3,2]$	1	$RM(2,2)$ $[4,4,1]$
$QRM_2(-1,-1)$ $[[4,0,1]]$ $S =$ $\{Z_0, Z_1, Z_2, Z_3\}$ codeword= $\|0\rangle\|0\rangle\|0\rangle\|0\rangle$ logicals= $\emptyset$		$QRM_2(0,0)$ $[[4,0,2]]$ $S =$ $\{Z_0Z_1, Z_1Z_2,$ Z_2Z_3, Z_3Z_0 $X_0X_1X_2X_3\}$ logicals= $\emptyset$		$QRM_2(1,1)$ $[[4,0,2]]$ $S =$ $\{Z_0Z_1Z_2Z_3,$ $X_0X_1, X_1X_2,$ $X_2X_3, X_3X_0\}$ logicals= $\emptyset$		$QRM_2(2,2)$ $[[4,0,1]]$ $S =$ $\{X_0, X_1, X_2, X_3\}$ codeword= $\|+\rangle\|+\rangle\|+\rangle\|+\rangle$ logicals= $\emptyset$
	$QRM_2(-1,0)$ $[[4,1,1]]$ $S =$ $\{Z_0Z_1, Z_1Z_2,$ $Z_2Z_3, Z_3Z_0\}$ logicals= $\{XXXX, Z, S, T, \ldots\}$		$QRM_2(0,1)$ $[[4,2,2]]$ $S =$ $\{Z_0Z_1Z_2Z_3,$ $X_0X_1X_2X_3\}$ logicals= $\{H, ZZ, SSSS, XX, \sqrt{X}\sqrt{X}\sqrt{X}\sqrt{X}\}$		$QRM_2(1,2)$ $[[4,1,1]]$ $S =$ $\{X_0X_1, X_1X_2,$ $X_2X_3, X_3X_0\}$ logicals= $\{ZZZZ, X, \sqrt{X}, \sqrt[4]{X}, \ldots\}$	
		$QRM_2(-1,1)$ $[[4,3,1]]$ $S =$ $\{Z_0Z_1Z_2Z_3\}$ logicals= $\{Z, ZZ, SS, SSSS, TTTT, XX, XXXX\}$		$QRM_2(0,2)$ $[[4,3,1]]$ $S =$ $\{X_0X_1X_2X_3\}$ logicals= $\{ZZ, ZZZZ, X, XX, \sqrt{X}\sqrt{X}, \sqrt{X}\sqrt{X}\sqrt{X}\sqrt{X}, \sqrt[4]{X}\sqrt[4]{X}\sqrt[4]{X}\sqrt[4]{X}\}$		
			$QRM_2(-1,2)$ $[[4,4,1]]$ $S = \{\}$ logicals= $\{H, Z, ZZ, ZZZZ, SSSS, X, XX, XXXX, \sqrt{X}\sqrt{X}\sqrt{X}\sqrt{X}\}$			

$QRM_{m=2}(q = 0, r = 1) \in [[n = 4, k = 2, d = 2]]$

The smallest quantum error detecting code

Codes-as-ansatz

Eight-qubit quantum error (correction) codes

$RM(-1,3)$ $[8,0,\infty]$	1	$RM(0,3)$ $[8,1,8]$	3	$RM(1,3)$ $[8,4,4]$	3	$RM(2,3)$ $[8,7,2]$	1	$RM(3,3)$ $[8,8,1]$
$QRM_3(-1,-1)$ $[[8,0,1]]$ $S: 8Z^{\otimes 1}$ codeword= $\vert 0\rangle^{\otimes 8}$ logicals= $\emptyset$		$QRM_3(0,0)$ $[[8,0,2]]$ $S:$ $12Z^{\otimes 2}, 1X^{\otimes 8}$ logicals= $\emptyset$		$QRM_3(1,1)$ $[[8,0,4]]$ $S:$ $6Z^{\otimes 4}, 6X^{\otimes 4}$ logicals= $\emptyset$		$QRM_3(2,2)$ $[[8,0,2]]$ $S:$ $1Z^{\otimes 8}, 12X^{\otimes 2}$ logicals= $\emptyset$		$QRM_3(3,3)$ $[[8,0,1]]$ $S: 8X^{\otimes 1}$ codeword= $\vert +\rangle^{\otimes 8}$ logicals= $\emptyset$
	$QRM_3(-1,0)$ $[[8,1,1]]$ $S: 12Z^{\otimes 2}$ logicals= $\{Z, S, T, \ldots\}$		$QRM_3(0,1)$ $[[8,3,2]]$ $S:$ $6Z^{\otimes 4}, 1X^{\otimes 8}$ logicals= $\{Z^{\otimes 2}, S^{\otimes 4}, T^{\otimes 8}\}$		$QRM_3(1,2)$ $[[8,3,2]]$ $S:$ $1Z^{\otimes 8}, 6X^{\otimes 4}$ logicals= $\{Z^{\otimes 4}\}$		$QRM_3(2,3)$ $[[8,1,1]]$ $S: 12X^{\otimes 2}$ logicals= $\{Z^{\otimes 8}\}$	
		$QRM_3(-1,1)$ $[[8,4,1]]$ $S: 6Z^{\otimes 4}$ logicals= $\{Z^{\otimes 1,2}, S^{\otimes 2,4}, T^{\otimes 4,8}\}$		$QRM_3(0,2)$ $[[8,6,2]]$ $S:$ $1Z^{\otimes 8}, 1X^{\otimes 8}$ logicals= $\{H, Z^{\otimes 2}, Z^{\otimes 4}, S^{\otimes 8}\}$		$QRM_3(1,3)$ $[[8,4,1]]$ $S: 6X^{\otimes 4}$ logicals= $\{Z^{\otimes 4}, Z^{\otimes 8}\}$		
			$QRM_3(-1,2)$ $[[8,7,1]]$ $S: 1\times 8Z$ logicals= $\{Z, Z^{\otimes 2}, Z^{\otimes 4}, S^{\otimes 4}, S^{\otimes 8}\}$		$QRM_3(0,3)$ $[[8,7,1]]$ $S: 1\times 8X$ logicals= $\{Z^{\otimes 2}, Z^{\otimes 4}, Z^{\otimes 8}\}$			
				$QRM_3(-1,3)$ $[[8,8,1]]$ $S=\{\}$ logicals= $\{H, Z, Z^{\otimes 2}, Z^{\otimes 4}, Z^{\otimes 8}, S^{\otimes 8}\}$				

16-qubit quantum error correction codes

$RM(-1,4)$		$RM(0,4)$		$RM(1,4)$		$RM(2,4)$		$RM(3,4)$		$RM(4,4)$
$[16,0,\infty]$	1	$[16,1,16]$	4	$[16,5,8]$	6	$[16,11,4]$	4	$[16,15,2]$	1	$[16,16,1]$

Row 1

- $QRM_4(-1,-1)$, $[[16,0,1]]$, S: $16Z^{\otimes 1}$, logicals= $\emptyset$
- $QRM_4(0,0)$, $[[16,0,2]]$, S: $32Z^{\otimes 2}, 1X^{\otimes 16}$, logicals= $\emptyset$
- $QRM_4(1,1)$, $[[16,0,4]]$, S: $24Z^{\otimes 4}, 8X^{\otimes 8}$, logicals= $\emptyset$
- $QRM_4(2,2)$, $[[16,0,4]]$, S: $8Z^{\otimes 8}, 24X^{\otimes 4}$, logicals= $\emptyset$
- $QRM_4(3,3)$, $[[16,0,2]]$, S: $1Z^{\otimes 16}, 32X^{\otimes 2}$, logicals= $\emptyset$
- $QRM_4(4,4)$, $[[16,0,1]]$, S: $16X^{\otimes 1}$, logicals= $\emptyset$

Row 2

- $QRM_4(-1,0)$, $[[16,1,1]]$, S: $32Z^{\otimes 2}$, logicals= $\{Z, S, T, \ldots\}$
- $QRM_4(0,1)$, $[[16,4,2]]$, S: $24Z^{\otimes 4}, 1X^{\otimes 16}$, logicals= $\{Z^{\otimes 2}, S^{\otimes 4}, T^{\otimes 8}, \sqrt{T}^{\otimes 16}\}$
- $QRM_4(1,2)$, $[[16,6,4]]$, S: $8Z^{\otimes 8}, 8X^{\otimes 8}$, logicals= $\{H, Z^{\otimes 4}, S^{\otimes 16}\}$
- $QRM_4(2,3)$, $[[16,4,2]]$, S: $1Z^{\otimes 16}, 24X^{\otimes 4}$, logicals= $\{Z^{\otimes 8}\}$
- $QRM_4(3,4)$, $[[16,1,1]]$, S: $32X^{\otimes 2}$, logicals= $\{Z^{\otimes 16}\}$

Row 3

- $QRM_4(-1,1)$, $[[16,5,1]]$, logicals= $\{Z^{\otimes 1,2}, S^{\otimes 2,4}, T^{\otimes 4,8}, \sqrt{T}^{\otimes 8,16}, \sqrt[4]{T}^{\otimes 16}\}$
- $QRM_4(0,2)$, $[[16,10,2]]$, logicals= $\{Z^{\otimes 2}, Z^{\otimes 4}, S^{\otimes 8}, S^{\otimes 16}\}$
- $QRM_4(1,3)$, $[[16,10,2]]$, logicals= $\{Z^{\otimes 4}, Z^{\otimes 8}\}$
- $QRM_4(2,4)$, $[[16,5,1]]$, logicals= $\{Z^{\otimes 8}, Z^{\otimes 16}\}$

Row 4

- $QRM_4(-1,2)$, $[[16,11,1]]$, logicals= $\{Z^{\otimes 1,2,4}, S^{\otimes 4,8,16}, T^{\otimes 16}\}$
- $QRM_4(0,3)$, $[[16,14,2]]$, logicals= $\{H, Z^{\otimes 2,4,8}, S^{\otimes 16}\}$
- $QRM_4(1,4)$, $[[16,11,1]]$, logicals= $\{Z^{\otimes 4}, Z^{\otimes 8}, Z^{\otimes 16}\}$

Row 5

- $QRM_4(-1,3)$, $[[16,15,1]]$, logicals= $\{Z^{\otimes 1,2,4,8}, S^{\otimes 8,16}\}$
- $QRM_4(0,4)$, $[[16,15,1]]$, logicals= $\{Z^{\otimes 2}, Z^{\otimes 4}, Z^{\otimes 8}, Z^{\otimes 16}\}$

Row 6

- $QRM_4(-1,4)$, $[[16,16,1]]$, logicals= $\{H, Z, Z^{\otimes 2}, Z^{\otimes 4}, Z^{\otimes 8}, Z^{\otimes 16}, S^{\otimes 16}\}$

$QRM_{m=3}(q=1, r=1) \in [[n=8, k=0, d=4]]$

Our first encounter with a quantum error correcting code

The geometric construction of quantum Reed-Muller codes

The duality of non-Clifford operations

What about non-Reed-Muller codes?

- Modified Reed-Muller codes. $[[7,1,3]]$, a code with transversal Cliffords.
- Non-CSS codes. $[[5,1,3]]$, the smallest qubit QECC

A periodic table of classical trit generalized Reed-Muller codes

			1 $GRM_3(-1,0)$ $[1,0,\infty]_3$	$GRM_3(0,0)$ $[1,1,1]_3$			
		1 $GRM_3(-1,1)$ $[3,0,\infty]_3$	1 $GRM_3(0,1)$ $[3,1,3]_3$	1 $GRM_3(1,1)$ $[3,2,2]_3$	$GRM_3(2,1)$ $[3,3,1]_3$		
	1 $GRM_3(-1,2)$ $[9,0,\infty]_3$	2 $GRM_3(0,2)$ $[9,1,9]_3$	3 $GRM_3(1,2)$ $[9,3,6]_3$	2 $GRM_3(2,2)$ $[9,6,3]_3$	1 $GRM_3(3,2)$ $[9,8,2]_3$	$GRM_3(4,2)$ $[9,9,1]_3$	
1 $GRM_3(-1,3)$ $[27,0,\infty]_3$	3 $GRM_3(0,3)$ $[27,1,27]_3$	6 $GRM_3(1,3)$ $[27,4,18]_3$	7 $GRM_3(2,3)$ $[27,10,9]_3$	6 $GRM_3(3,3)$ $[27,17,6]_3$	3 $GRM_3(4,3)$ $[27,23,3]_3$	1 $GRM_3(5,3)$ $[27,26,2]_3$	$GRM_3(6,3)$ $[27,27,1]_3$
Trivial codes	Repetition codes					Single parity-check codes	Universe codes

One-qutrit quantum (stabilized) states

$GRM(-1,0)$ $[1,0,\infty]_3$	1	$GRM(0,0)$ $[1,1,1]_3$
$QGRM_0^3(-1,-1)$ $[[1,0,1]]_3$ $S=\{Z=\begin{bmatrix}1&0&0\\0&\omega&0\\0&0&\omega^2\end{bmatrix}\}$ codeword$=\|0\rangle=\begin{bmatrix}1\\0\\0\end{bmatrix}$ logicals$=\emptyset$		$QGRM_0^3(0,0)$ $[[1,0,1]]_3$ $S=\{X=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\}$ codeword$=\|+\rangle=\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$ logicals$=\emptyset$
	$QGRM_0^3(-1,0)$ $[[1,1,1]]_3$ $S=\{\}$ codeword$=\|\psi\rangle$ logicals$=U$	

Three-qutrit quantum error (detection) codes

$GRM_3(-1,1)$ $[3,0,\infty]_3$	1	$GRM_3(0,1)$ $[3,1,3]_3$	1	$GRM_3(1,1)$ $[3,2,2]_3$	1	$GRM_3(2,1)$ $[3,3,1]_3$
$QGRM_1^3(-1,-1)$ $[[3,0,1]]_3$ $S=$ $\{Z_0, Z_1, Z_2\}$ codeword= $\|0\rangle \|0\rangle \|0\rangle$ logicals= $\emptyset$		$QGRM_1^3(0,0)$ $[[3,0,2]]_3$ $S=$ $\{Z_0Z_1, Z_1Z_2, Z_2Z_0$ $X_0X_1X_2\}$ logicals= $\emptyset$		$QGRM_1^3(1,1)$ $[[3,0,2]]_3$ $S=\{Z_0Z_1Z_2,$ $X_0X_1, X_1X_2, X_2X_0\}$ logicals= $\emptyset$		$QGRM_1^3(2,2)$ $[[3,0,1]]_3$ $S=$ $\{X_0, X_1, X_2\}$ codeword= $\|+\rangle \|+\rangle \|+\rangle$ logicals= $\emptyset$
	$QGRM_1^3(-1,0)$ $[[3,1,1]]_3$ $S=$ $\{Z_0Z_1, Z_1Z_2, Z_2Z_0\}$ logicals= $\{XXX, Z, S, T, \ldots\}$		$QGRM_1^3(0,1)$ $[[3,1,2]]_3$ $S=\{Z_0Z_1Z_2,$ $X_0X_1X_2\}$ logicals= $\{H, ZZ, SSS, XX, \sqrt{X}\sqrt{X}\sqrt{X}\}$		$QGRM_1^3(1,2)$ $[[3,1,1]]_3$ $S=$ $\{X_0X_1, X_1X_2, X_2X_0\}$ logicals= $\{ZZZ, X, \sqrt{X}, \sqrt[4]{X}, \ldots\}$	
		$QGRM_1^3(-1,1)$ $[[3,2,1]]_3$ $S=\{Z_0Z_1Z_2\}$ logicals= $\{Z, ZZ, SS, SSS, XX, XXX\}$		$QGRM_1^3(0,2)$ $[[3,2,1]]_3$ $S=\{X_0X_1X_2\}$ logicals= $\{ZZ, ZZZ, X, XX, \sqrt{X}\sqrt{X}, \sqrt{X}\sqrt{X}\sqrt{X}\}$		
			$QGRM_1^3(-1,2)$ $[[3,3,1]]_3$ $S=\{\}$ logicals= $\{H, Z, ZZ, ZZZ, SSS, X, XX, XXX, \sqrt{X}\sqrt{X}\sqrt{X}\}$			

Nine-qutrit quantum error (correction) codes

$GRM_3(-1,2)$ $[9,0,\infty]_3$	1	$GRM_3(0,2)$ $[9,1,9]_3$	2	$GRM_3(1,2)$ $[9,3,6]_3$	3	$GRM_3(2,2)$ $[9,6,3]_3$	2	$GRM_3(3,2)$ $[9,8,2]_3$	1	$GRM_3(4,2)$ $[9,9,1]_3$

$QGRM_2^3(-1,-1)$ $[[9,0,1]]_3$ $S=$ $9Z^{\otimes 1}$ logicals= $\emptyset$	$QGRM_2^3(0,0)$ $[[9,0,2]]_3$ $S=$ $?Z^{\otimes 2}, 1X^{\otimes 9}$ logicals= $\emptyset$	$QGRM_2^3(1,1)$ $[[9,0,3]]_3$ $S=$ $6Z^{\otimes 3}, ?X^{\otimes 6}$ logicals= $\emptyset$	$QGRM_2^3(2,2)$ $[[9,0,3]]_3$ $S=$ $?Z^{\otimes 6}, 6X^{\otimes 3}$ logicals= $\emptyset$	$QGRM_2^3(3,3)$ $[[9,0,2]]_3$ $S=$ $1Z^{\otimes 9}, ?X^{\otimes 2}$ logicals= $\emptyset$	$QGRM_2^3(4,4)$ $[[9,0,1]]_3$ $S=$ $9X^{\otimes 1}$ logicals= $\emptyset$

$QGRM_2^3(-1,0)$ $[[9,1,1]]_3$ $S=$ $?Z^{\otimes 2}$ logicals= $\{Z, S, T, \ldots\}$	$QGRM_2^3(0,1)$ $[[9,2,2]]_3$ $S=$ $6Z^{\otimes 3}, 1X^{\otimes 9}$ logicals= $\{Z^{\otimes 2}, S^{\otimes 3}, T^{\otimes 9}\}$	$QGRM_2^3(1,2)$ $[[9,3,3]]_3$ $S=$ $?Z^{\otimes 6}, ?X^{\otimes 6}$ logicals= $\{H, Z^{\otimes 3}, S^{\otimes 9}\}$	$QGRM_2^3(2,3)$ $[[9,2,2]]_3$ $S=$ $1Z^{\otimes 9}, 6X^{\otimes 3}$ logicals= $\{Z^{\otimes 6}\}$	$QGRM_2^3(3,4)$ $[[9,1,1]]_3$ $S=$ $?X^{\otimes 2}$ logicals= $\{Z^{\otimes 9}\}$

$QGRM_2^3(-1,1)$ $[[9,3,1]]_3$ logicals= $\{Z^{\otimes 1,2}, S^{\otimes 2,3}, T^{\otimes 6,9}\}$	$QGRM_2^3(0,2)$ $[[9,5,2]]_3$ logicals= $\{Z^{\otimes 2,3}, S^{\otimes 6,9}\}$	$QGRM_2^3(1,3)$ $[[9,5,2]]_3$ logicals= $\{Z^{\otimes 3,6}\}$	$QGRM_2^3(2,4)$ $[[9,3,1]]_3$ logicals= $\{Z^{\otimes 6,9}\}$

$QGRM_2^3(-1,2)$ $[[9,6,1]]_3$ logicals= $\{Z^{\otimes 1,2,3}, S^{\otimes 3,6,9}\}$	$QGRM_2^3(0,3)$ $[[9,7,2]]_3$ logicals= $\{H, Z^{\otimes 2,3,6}, S^{\otimes 9}\}$	$QGRM_2^3(1,4)$ $[[9,6,1]]_3$ logicals= $\{Z^{\otimes 3,6,9}\}$

$QGRM_2^3(-1,3)$ $[[9,8,1]]_3$ logicals= $\{Z^{\otimes 1,2,3,6}, S^{\otimes 6,9}\}$	$QGRM_2^3(0,4)$ $[[9,8,1]]_3$ logicals= $\{Z^{\otimes 2,3,6,9}\}$

$QGRM_2^3(-1,4)$ $[[9,9,1]]_3$ logicals= $\{H, Z^{\otimes 1,2,3,6,9}, S^{\otimes 9}\}$

A periodic table of classical quart generalized Reed-Muller codes

1

$GRM_4(-1, 0)$ $[1, 0, \infty]_4$ | $GRM_4(0, 0)$ $[1, 1, 1]_4$

1 1 1 1

$GRM_4(-1, 1)$ $[4, 0, \infty]_4$ | $GRM_4(0, 1)$ $[4, 1, 4]_4$ | $GRM_4(1, 1)$ $[4, 2, 3]_4$ | $GRM_4(2, 1)$ $[4, 3, 2]_4$ | $GRM_4(3, 1)$ $[4, 4, 1]_4$

1 2 3 4 3 2 1

$GRM_4(-1, 2)$ $[16, 0, \infty]_4$ | $GRM_4(0, 2)$ $[16, 1, 16]_4$ | $GRM_4(1, 2)$ $[16, 3, 12]_4$ | $GRM_4(2, 2)$ $[16, 6, 8]_4$ | $GRM_4(3, 2)$ $[16, 10, 4]_4$ | $GRM_4(4, 2)$ $[16, 13, 3]_4$ | $GRM_4(5, 2)$ $[16, 15, 2]_4$ | $GRM_4(6, 2)$ $[16, 16, 1]_4$

Trivial codes

Repetition codes

Self-dual code

Single parity-check codes

Universe codes