Sdim: A Simulator for High-Dimensional Quantum Fault Tolerance

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Based on work with

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Bart Soree and **James Keppens** (KU Leuven)

The setting

Quantum computing is an exciting topic in computing and physics for a myriad of reasons.

- Algorithms (e.g. Shor's algorithm)
- Communication (e.g. QKD)
- Basic science (e.g. quantum matter)
- Engineering (e.g. materials)

The reality

Quantum computers are noisy beasts.

- Scaling the *number of qubits* is **hard**.
- Improving *coherence time* is **hard**.
- Suppressing *sources of noise* in hardware is **hard**.
- Practically dealing with *circuit level noise* is **hard**.

Qudits: what are they?

Qubit: 2-dim (2-level) quantum system, measurement yields **bits**

Qu**dit**: *d*-level quantum system, measurement yields integers mod *d*

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, |d-1\rangle = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

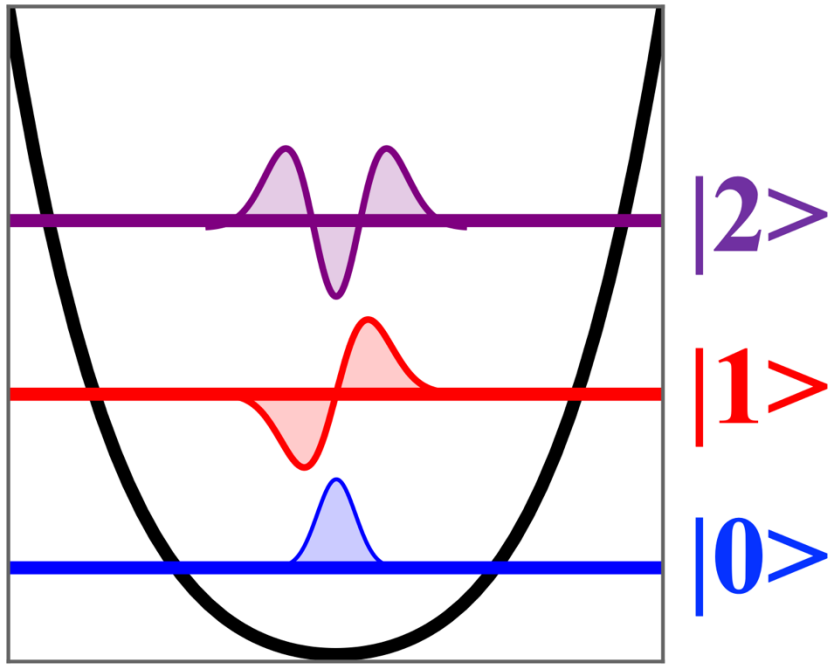

Image source : https://arxiv.org/abs/2212.05054

Qudits at Fermilab

SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

Qudit-based quantum computing with SRF cavities at Fermilab

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Roadmap

Quantum Simulation for High-Energy Physics

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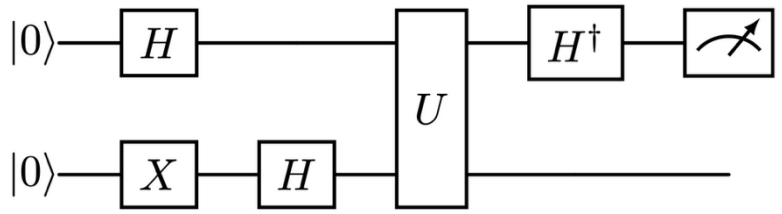
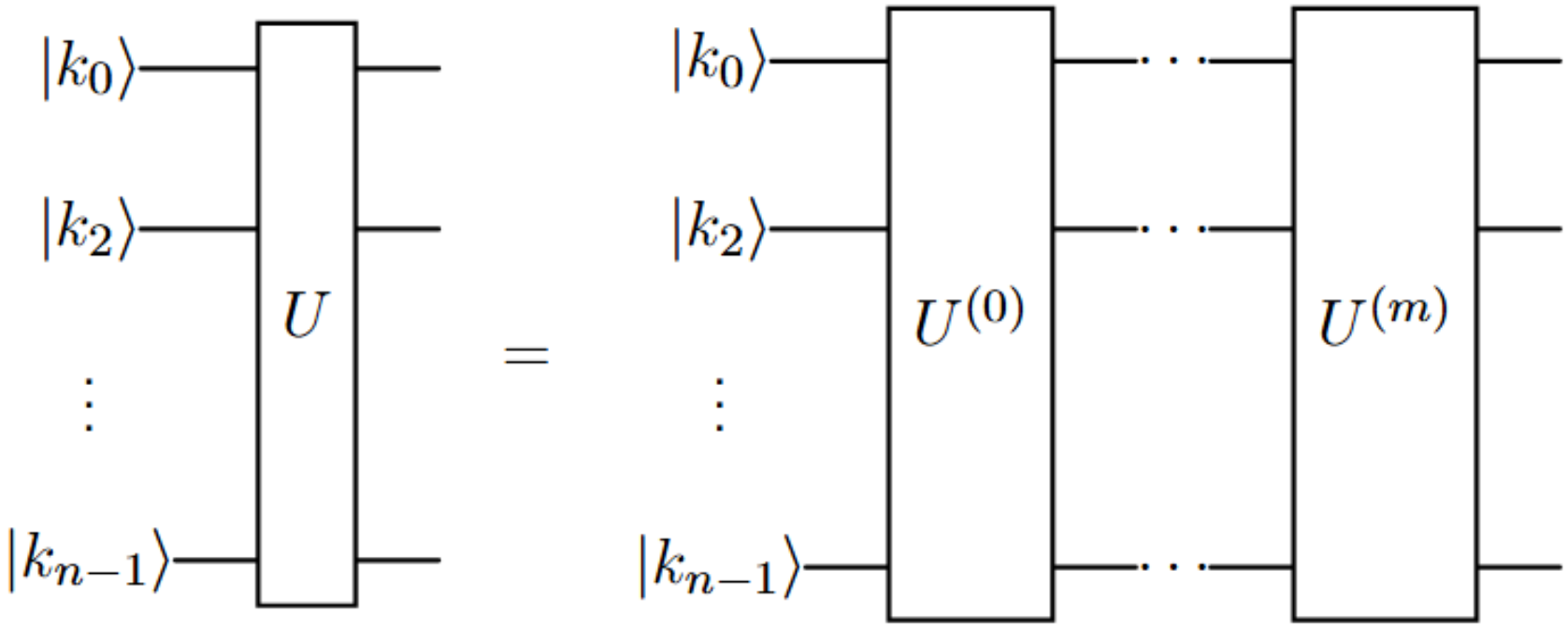
Sdim: A Qudit Stabilizer Simulator

```python
is_constant = random.choice(["True", "False"])
c = Circuit(2, d)  # Initial circuit is |00>
c.add_gate("H", 0) # F |0> on qudit 0
c.add_gate("X", 1) # Flip ancilla |0> to |1>
c.add_gate("H", 1) # F |1> on ancilla
if is_constant:
    function_constant = random.choice([j for j in range(d)])
    for i in range(function_constant):
        c.add_gate("X", 1) # Oracle for f=j
else:
    c.add_gate("CNOT", 0, 1) # Oracle f=id
c.add_gate("H_INV", 0) # F^t |0> on qudit 0
c.add_gate("M", 0) # Measure qudit 0
expected = 0 if is_constant else d - 1
p = Program(c)
assert p.simulate() == [MeasurementResult(0, True, expected)]
```



- Minimal Python library for a simple circuit abstraction
- High-dimensional stabilizer circuits tooled for error correction research (e.g. detector-error models)
- Efficient Pauli frame sampler
- `pip install sdim`

Circuits



A *depth* m *circuit* is $U|\psi\rangle$, where $U = \Pi_{i=0}^{m} U^{(i)} = U^{(m)} U^{(m-1)} \dots U^{(0)}$

Different representations of quantum info

- Statevector: state is a **2^n** vector, gates are **2^n** x **2^n** matrices

$$\begin{bmatrix} U_{0,0} & U_{0,1} & \cdots & \cdots & U_{0,2^n-2} & U_{0,2^n-1} \\ U_{1,0} & U_{1,1} & \cdots & \cdots & U_{1,2^n-2} & U_{1,2^n-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ U_{2^n-2,0} & U_{2^n-2,1} & \cdots & \cdots & U_{2^n-2,2^n-2} & U_{2^n-2,2^n-1} \\ U_{2^n-1,0} & U_{2^n-1,1} & \cdots & \cdots & U_{2^n-1,2^n-2} & U_{2^n-1,2^n-1} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ \vdots \\ s_{2^n-2} \\ s_{2^n-1} \end{bmatrix}$$

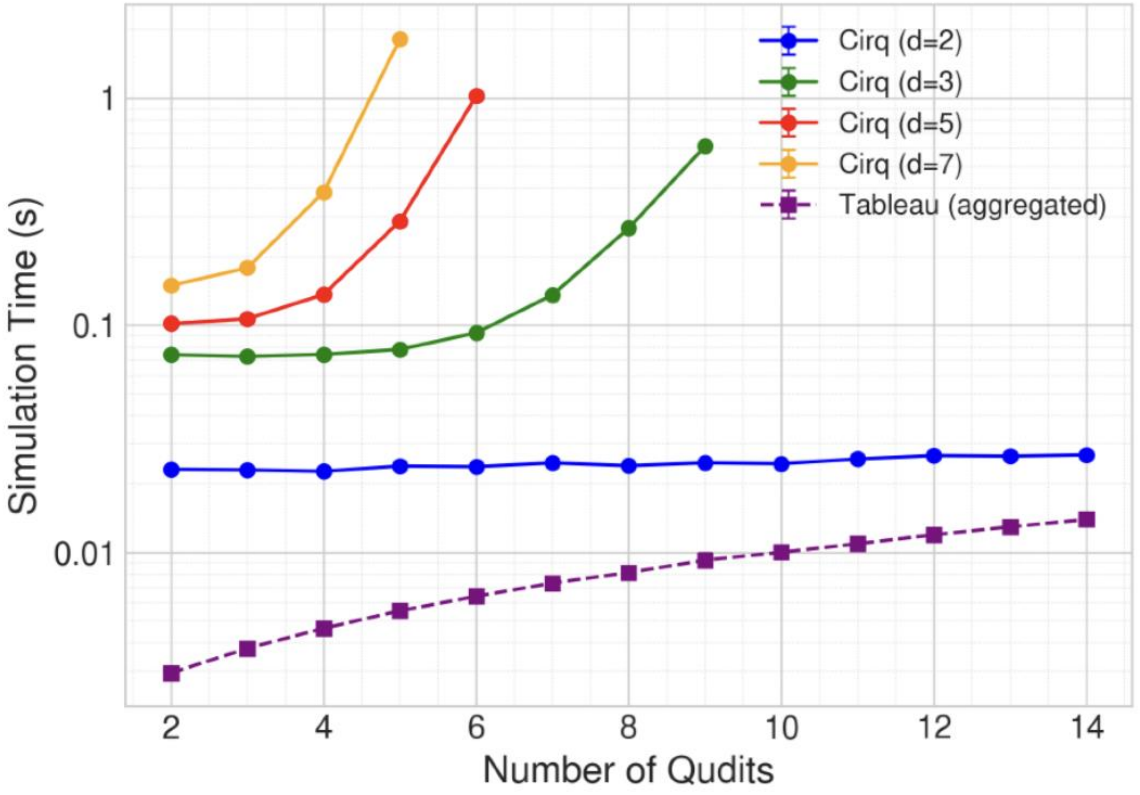
U can be {*CNOT, H, P, T*}

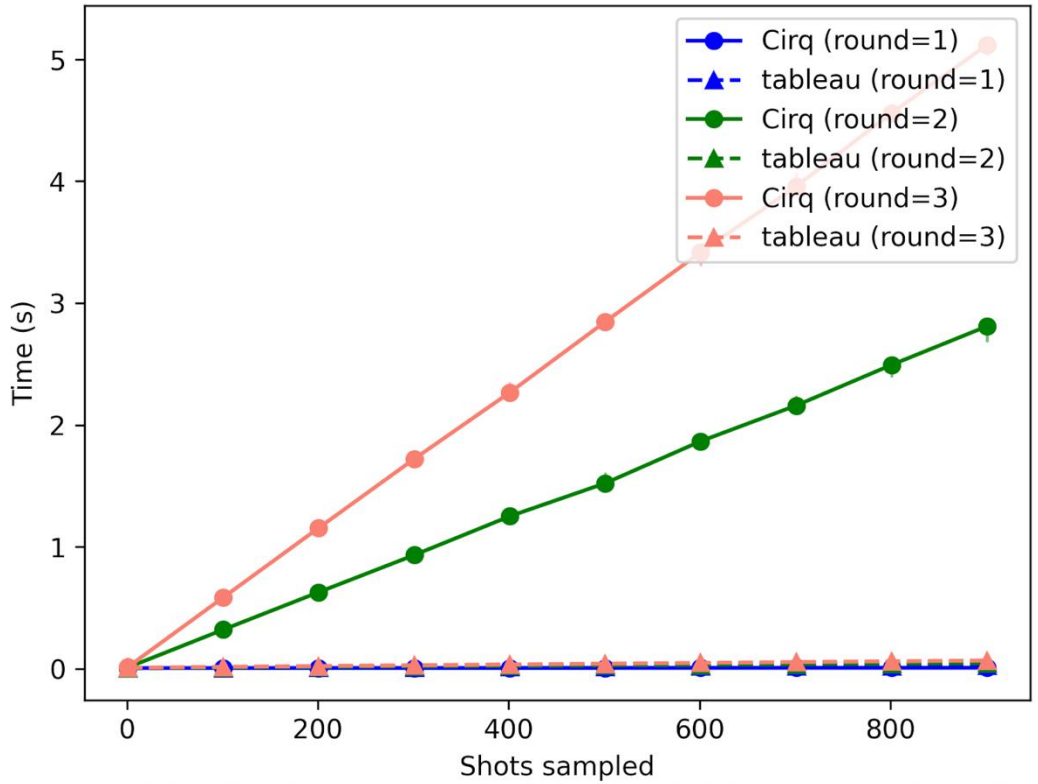
- Tableau : state is a **2n** x **2n** table of integers, gates are simple row-col **rewrite rules.**

$$\left[\begin{array}{ccc|ccc|c} x_{0,0} & \cdots & x_{0,n-1} & z_{0,0} & \cdots & z_{0,n-1} & r_0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n-1,0} & \cdots & x_{n-1,n-1} & z_{n,0} & \cdots & z_{n-1,n-1} & r_{n-1} \\ \hline x_{n,0} & \cdots & x_{n,n} & z_{n,0} & \cdots & z_{n,n-1} & r_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{2n-1,1} & \cdots & x_{2n-1,n-1} & z_{2n-1,0} & \cdots & z_{2n-1,n-1} & r_{2n-1} \end{array}\right]$$

Tradeoff: *U* can only be {*CNOT, H, P*}

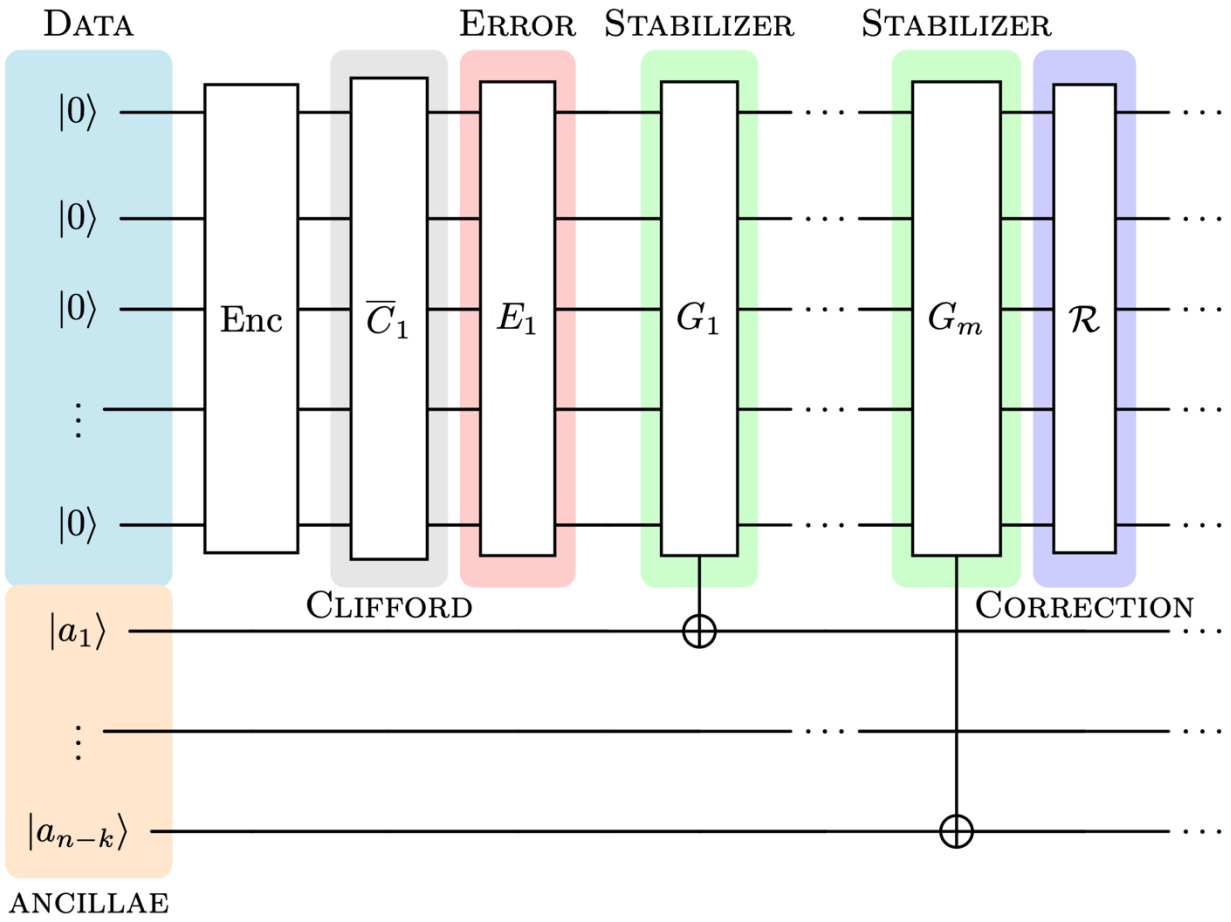
Speedup from statevector simulation





Stabilizer states trade universality for ***exponential*** speedup!

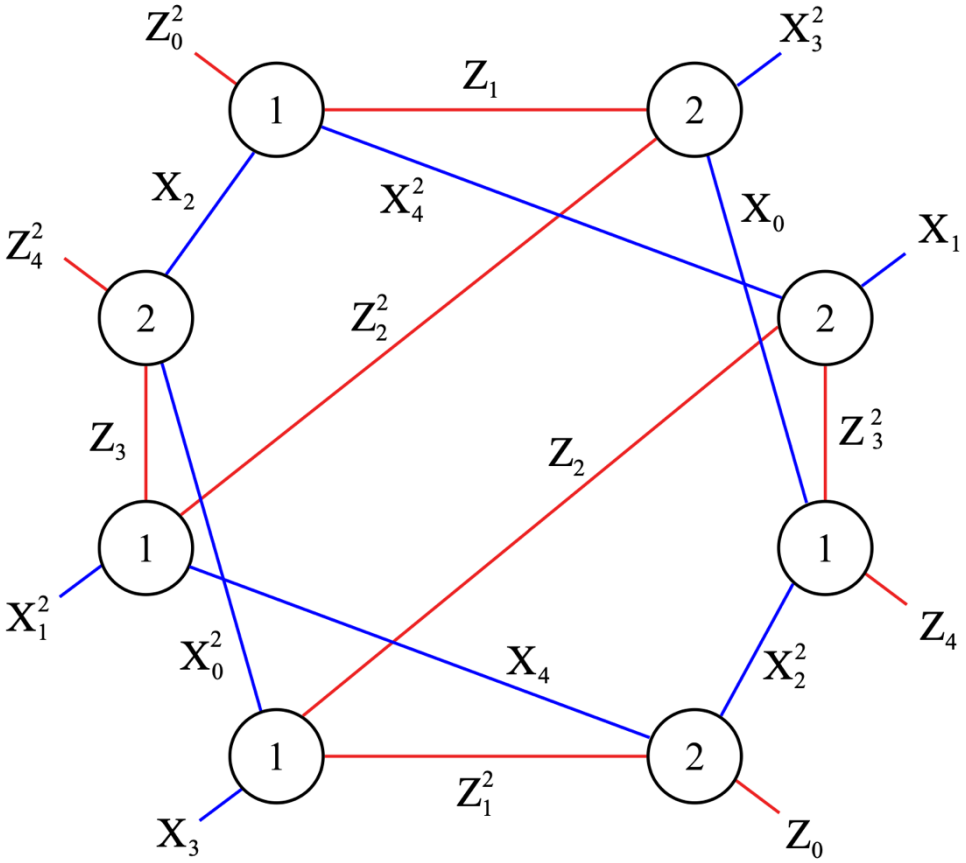
What is a code?



A quantum code is a protocol that turns **quantum info** (the error) into **classical info** (integers).

It turns a space of ***n*** **physical qudits** into a space of ***k*** **logical qudits. Errors do not depend on the state.**

Decoding and thresholds

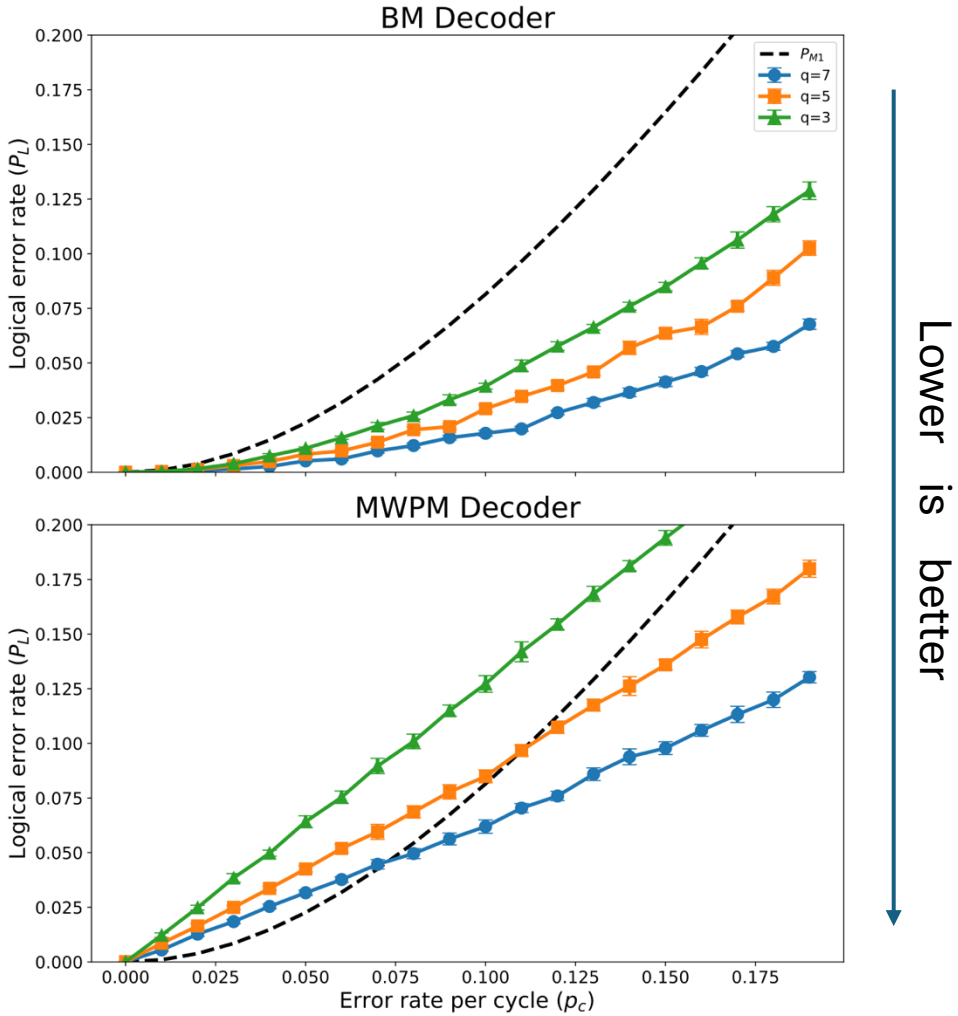


In practice, it takes too much time/space to determine exact corrections. We transform codes into **graphs that can efficiently approximate the correction.**

Threshold: physical error rates that a code-protected system can successfully tolerate.

Image source : https://arxiv.org/abs/2502.05992

Some results with a 5-qutrit code



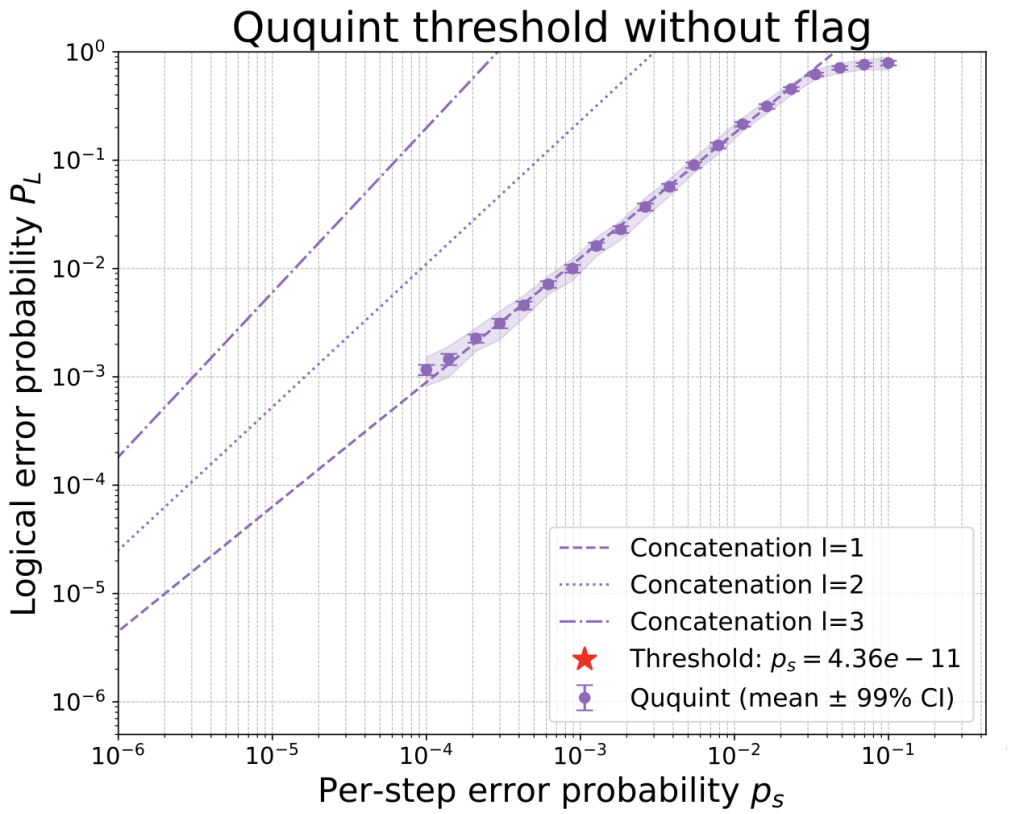
The $[[5,1,3]]_q$ code has the following stabilizers:

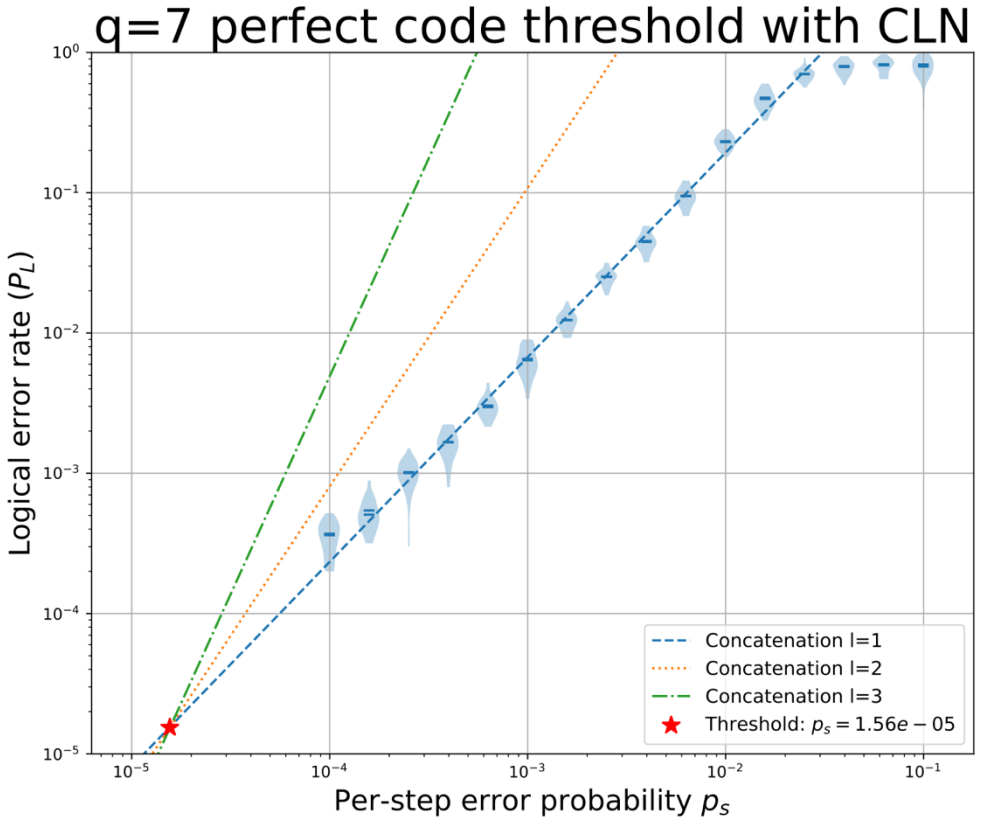
$$G_1 = I \otimes Z \otimes X \otimes X^\dagger \otimes Z^\dagger$$
$$G_2 = Z^\dagger \otimes I \otimes Z \otimes X \otimes X^\dagger$$
$$G_3 = X^\dagger \otimes Z^\dagger \otimes I \otimes Z \otimes X$$
$$G_4 = X \otimes X^\dagger \otimes Z^\dagger \otimes I \otimes Z$$

TABLE I: Times required to sample syndrome measurements with the noisy $q = 7$ five-qudit codes in Sdim and in Cirq.

	Sdim	Cirq
LER sample	$9.9 \cdot 10^{-5}$ s	66 s
Threshold sample	$2.3 \cdot 10^{-4}$ s	–

Some results with a 5-qutrit code (cont.)





Lower error rate is better

$$P_L^1(p_s) = ap_s^b \quad , \quad P_L^k = P_L^1(p_s) \circ P_L^{k-1}$$

Higher threshold is better

Image source : https://arxiv.org/abs/2502.05992

Outlook

- Examine qudit **qLDPC** codes

- Conduct extensive simulations with experimentally relevant error parameters and conditions

Sdim: A qudit stabilizer simulator on arXiv

Thanks for listening!